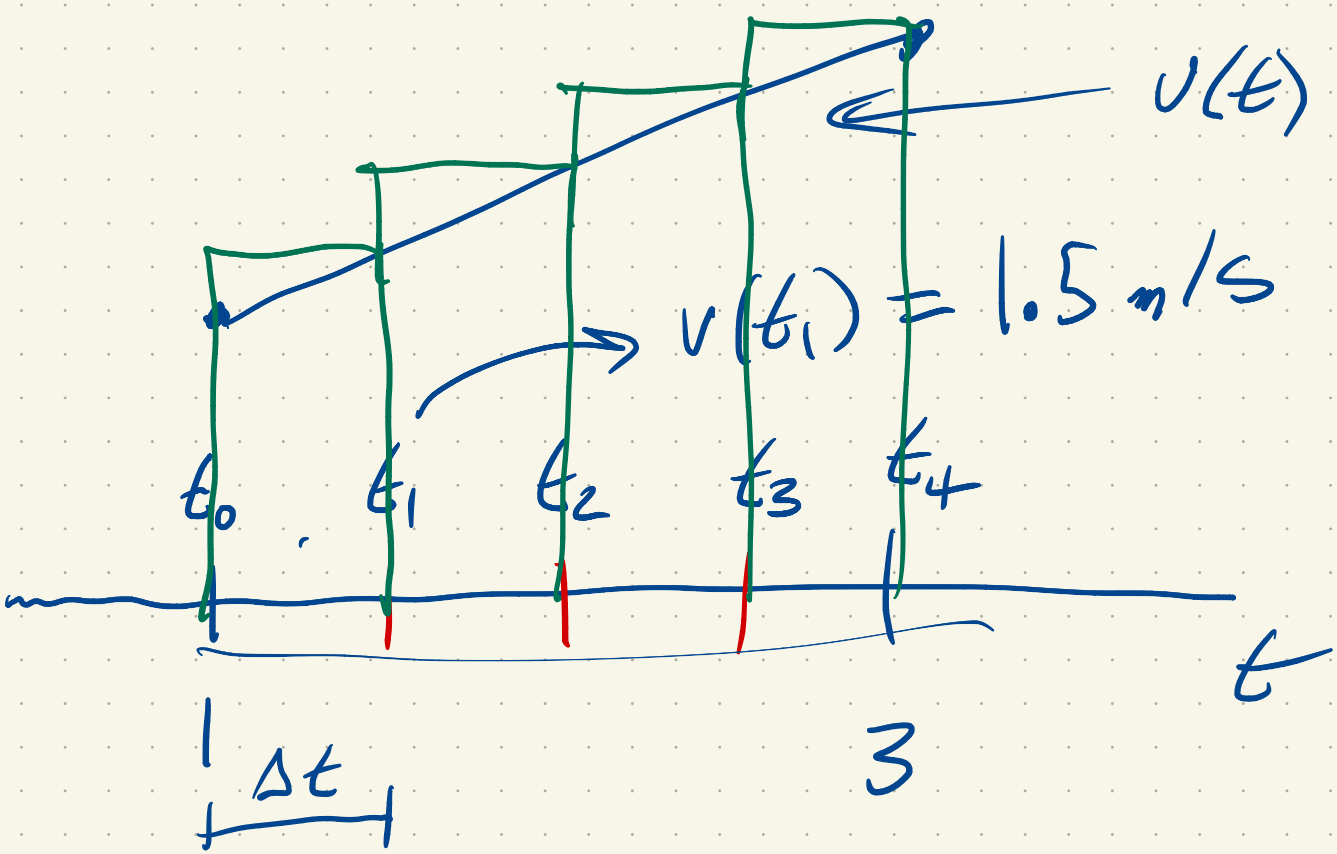
Suppose a vole is running

$$v(t) = t \text{ m/s} \qquad 1 \le t \le 3$$

What distance did the vole travel?



$$\Delta t = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2} \quad (s) \qquad v(t) = t$$

$$t_0 = 1 \qquad v(t_0) = t_0 = 1$$

$$t_1 = 1 + \Delta t = 1 + \frac{1}{2} \qquad v(t_1) = t_1 = 1 + \frac{1}{2}$$

$$t_2 = 1 + 2\Delta t = 1 + \frac{2}{2}$$

$$t_3 = 1 + 3\Delta t = 1 + \frac{3}{2}$$

$$t_4 = 1 + 4\Delta t = 1 + \frac{4}{2} = 3 \qquad v(t_4) = t_4 = 3$$

$$d_1 = v(t_1) \cdot \Delta t \qquad \left(1.5 \cdot \frac{1}{2} = 0.75\ m\right)$$

$$d_2 = v(t_2) \cdot \Delta t$$

$$d_3 = v(t_3) \cdot \Delta t$$

$$d_4 = v(t_4) \cdot \Delta t = 3 \cdot \frac{1}{2} = \frac{3}{2}\ m$$

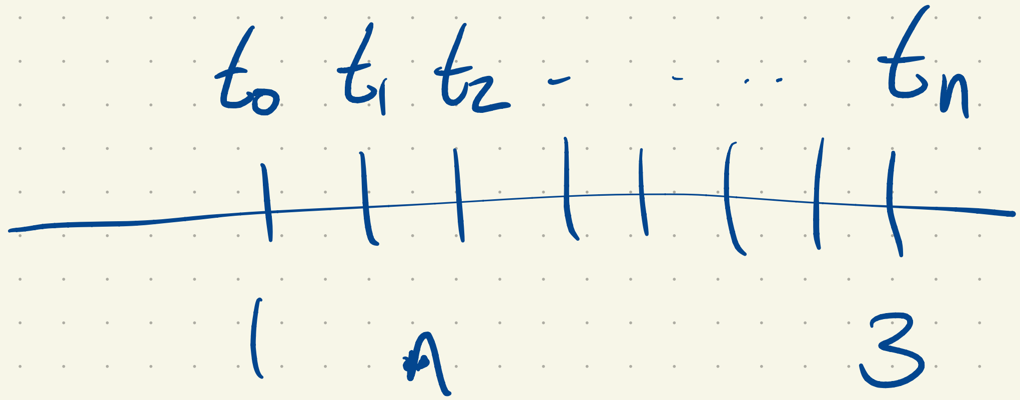
TBA

Estimated distance: $d_1 + d_2 + d_3 + d_4 = \boxed{}\ m$

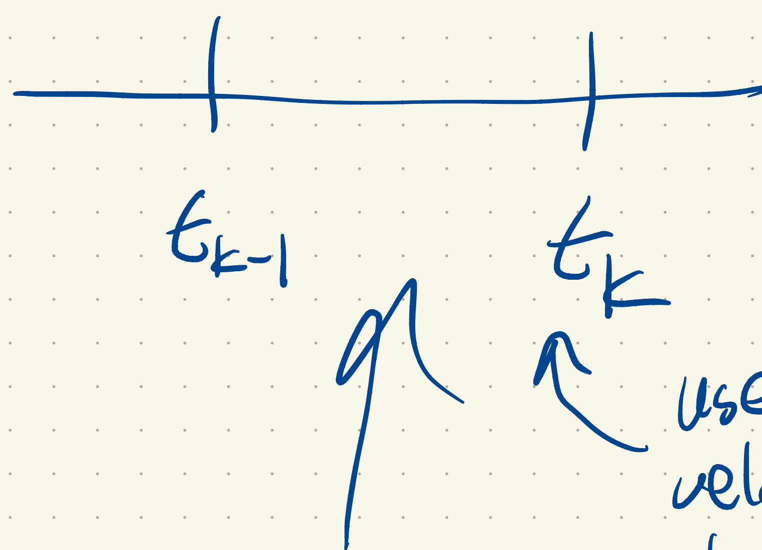
$v(t) = t$

n subintervals

$v(t_1) = t_1$



$$\Delta t = \frac{3-1}{n} = \frac{2}{n}$$



$t_0 = 1$

$t_1 = 1 + \Delta t$

$t_k = 1 + k\Delta t$

use velocity here

velocity on $[t_{k-1}, t_k]$ is

$$v(t_k)$$

distance traveled over $[t_{k-1}, t_k]$ is

$$v(t_k)\Delta t$$

Total distance traveled:

$$v(t_1)\Delta t + v(t_2)\Delta t + \cdots + v(t_n)\Delta t$$

$$R_n = \sum_{k=1}^{n} v(t_k)\Delta t$$

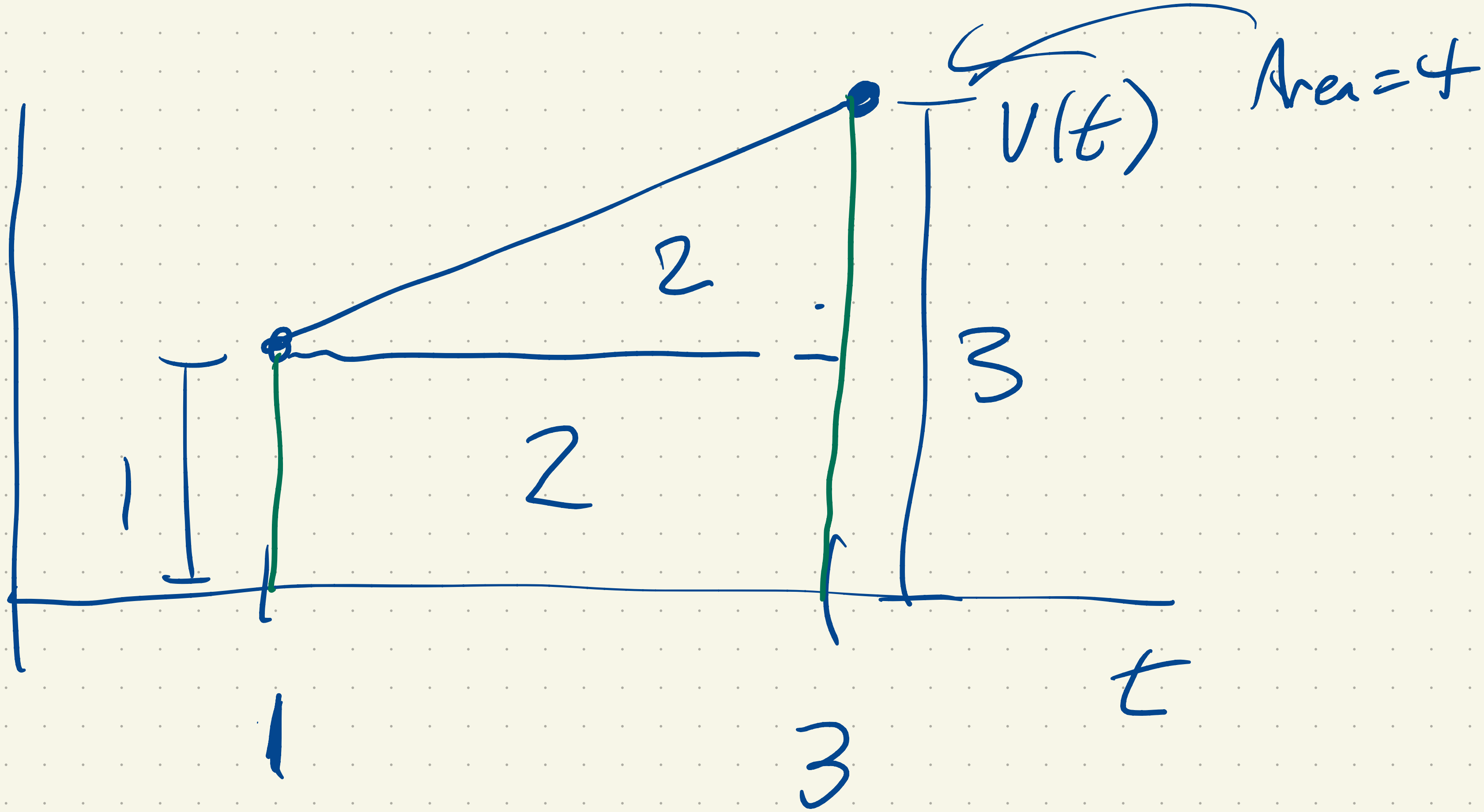
Riemann Sum

$$v(t) = t, \quad t_k = 1 + k\Delta t$$

$$R_n = \sum_{k=1}^{n} [1 + k\Delta t]\Delta t$$

$$\Delta t = \frac{3-1}{7}$$

$$= \sum_{k=1}^{n} [\Delta t + k\Delta t^2]$$



$$\sum_{k=1}^{n}\left[\Delta t + k\,\Delta t^2\right]$$

$$\sum_{k=1}^{n}\Delta t + \sum_{k=1}^{n} k\,\Delta t^2$$

$$\Delta t = \frac{2}{n}$$

$$\underbrace{\frac{2}{n}\sum_{k=1}^{n} 1} + \left(\frac{2}{n}\right)^2 \sum_{k=1}^{n} k$$

$$\underbrace{\frac{2}{n}\cdot n}$$

$$2$$

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n$$

$$1 + 2 + 3 + \cdots + 100$$

$$100 + 99 + 98 + \cdots + 1$$

$$\underbrace{101 + 101 + 101 + \cdots + 101}_{100}$$

$$1 + 2 + \cdots + 100 = \frac{100 \cdot (101)}{2}$$