L'Hopital's Rule

$$\frac{0}{0} \rightarrow \frac{\Delta x}{\Delta t}$$

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

$$\lim_{x \to 0} \frac{\sin(x)}{x} = \lim_{x \to 0} \frac{\frac{d}{dx}\sin(x)}{\frac{d}{dx}x} = \lim_{x \to 0} \frac{\cos(x)}{1}$$

$$\frac{0}{0}$$

$$= \frac{\cos(0)}{1} = 1$$

L'Hopital's Rule:

$$\lim_{x \to a} \frac{f(x)}{g(x)} \qquad f(a) = 0$$

$$g(a) = 0$$

$$\hookrightarrow = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

$$\lim_{x \to 2\pi} \frac{\cos(x) - 1}{x - 2\pi} = \lim_{x \to 2\pi} \frac{-\sin(x)}{1} = \frac{-\sin(2\pi)}{1} = \frac{0}{1}$$

$$= 0$$

$$\frac{0}{0}$$

1) This works for $\frac{\infty}{\infty}$ as well as $\frac{0}{0}$

2) This applies to limits at ∞.

$$\lim_{x\to\infty} \frac{x}{e^x} = \lim_{x\to\infty} \frac{1}{e^x} = \lim_{x\to\infty} e^{-x} = 0$$

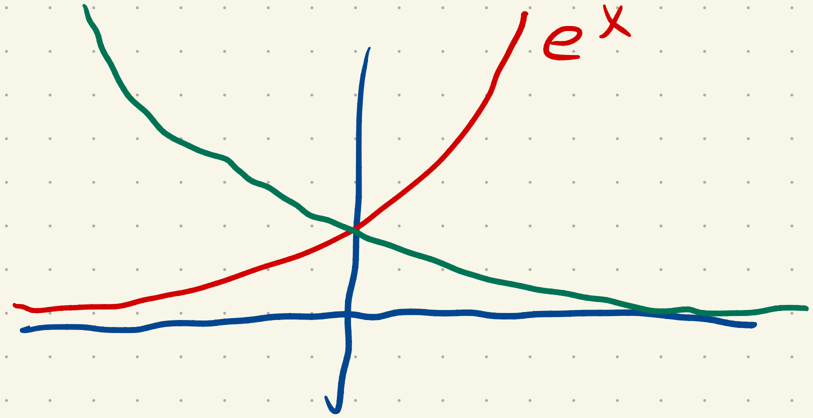
$\frac{\infty}{\infty}$

3) It applies to $0 \cdot \infty$ if can massage

it into $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

e.g. $\lim_{x \to \infty} x^3 e^{-x}$

$\infty \cdot 0$



$$\lim_{x \to \infty} \frac{x^3}{e^x} \overset{\frac{\infty}{\infty}}{=} \lim_{x \to \infty} \frac{3x^2}{e^x}$$

$$\overset{\frac{\infty}{\infty}}{=} \lim_{x \to \infty} \frac{6x}{e^x}$$

$$\overset{\frac{\infty}{\infty}}{=} \lim_{x \to \infty} \frac{6}{e^x}$$

$$= \frac{6}{\infty} = 0$$

---

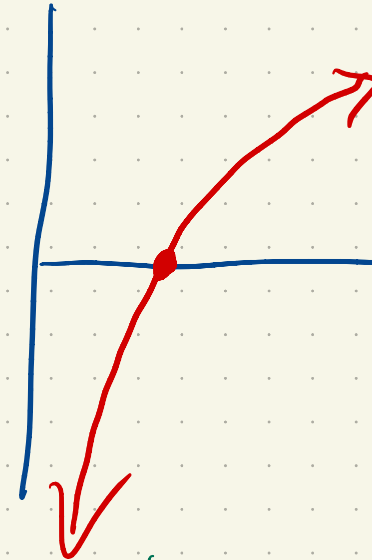
4) This also works for:

$$1^\infty \quad 0^0 \quad \infty^0$$

using some ln / exp tricks

$$\lim_{x \to 0^+} x^x \qquad 0^0$$

a) $\displaystyle \lim_{x\to 0^+} \ln\left(x^x\right) = \lim_{x\to 0^+} x\ln(x)$



$0\cdot(-\infty)$

$$= \lim_{x\to 0^+} \frac{\ln(x)}{1/x} \qquad \frac{-\infty}{\infty}$$

$$= \lim_{x\to 0^+} \frac{1/x}{-1/x^2}$$

$$= \lim_{x\to 0^+} -\frac{x^2}{x} = \lim_{x\to 0^+} -x = 0$$

b) $x^x = e^{\ln(x^x)}$

$$\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{\ln(x^x)} = e^0 = 1$$

$$\boxed{\lim_{x\to 0^+} x^x = 1}$$