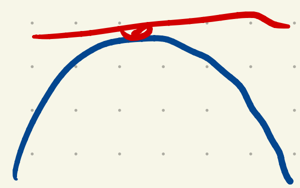
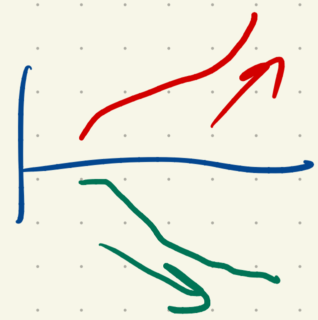
Last class



local extrema (sigh)

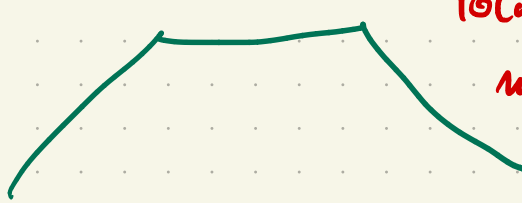
On an interval

- $f'(x) = 0 \Rightarrow$ constant
- $f'(x) > 0 \Rightarrow$ (strictly) increasing
- $f'(x) < 0 \Rightarrow$ (strictly) decreasing



First Derivative Test

$f'(x)$ + | − (at c)

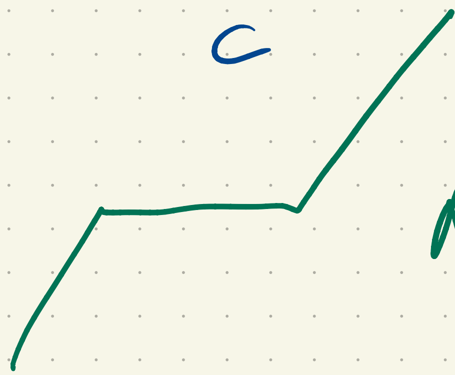


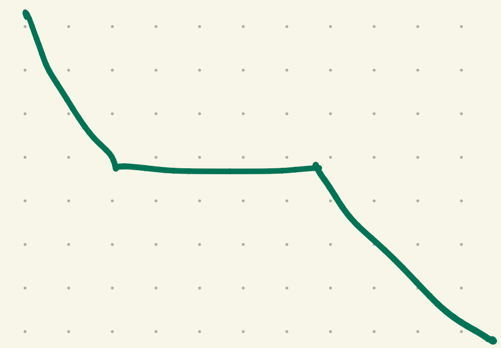
local max

− | + (at c)

local min

$f'(x)$ $+$ c $+$ $-$ c $-$



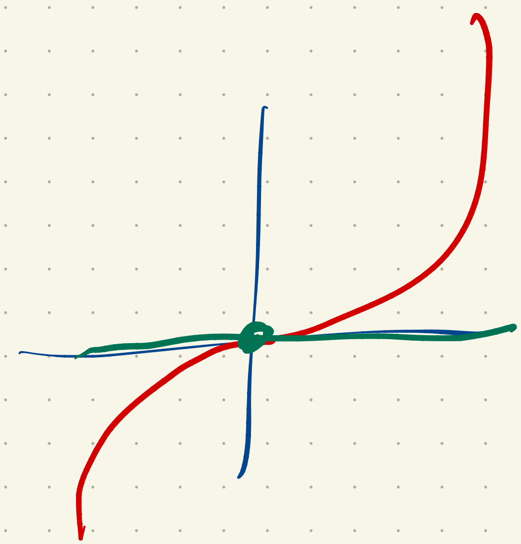


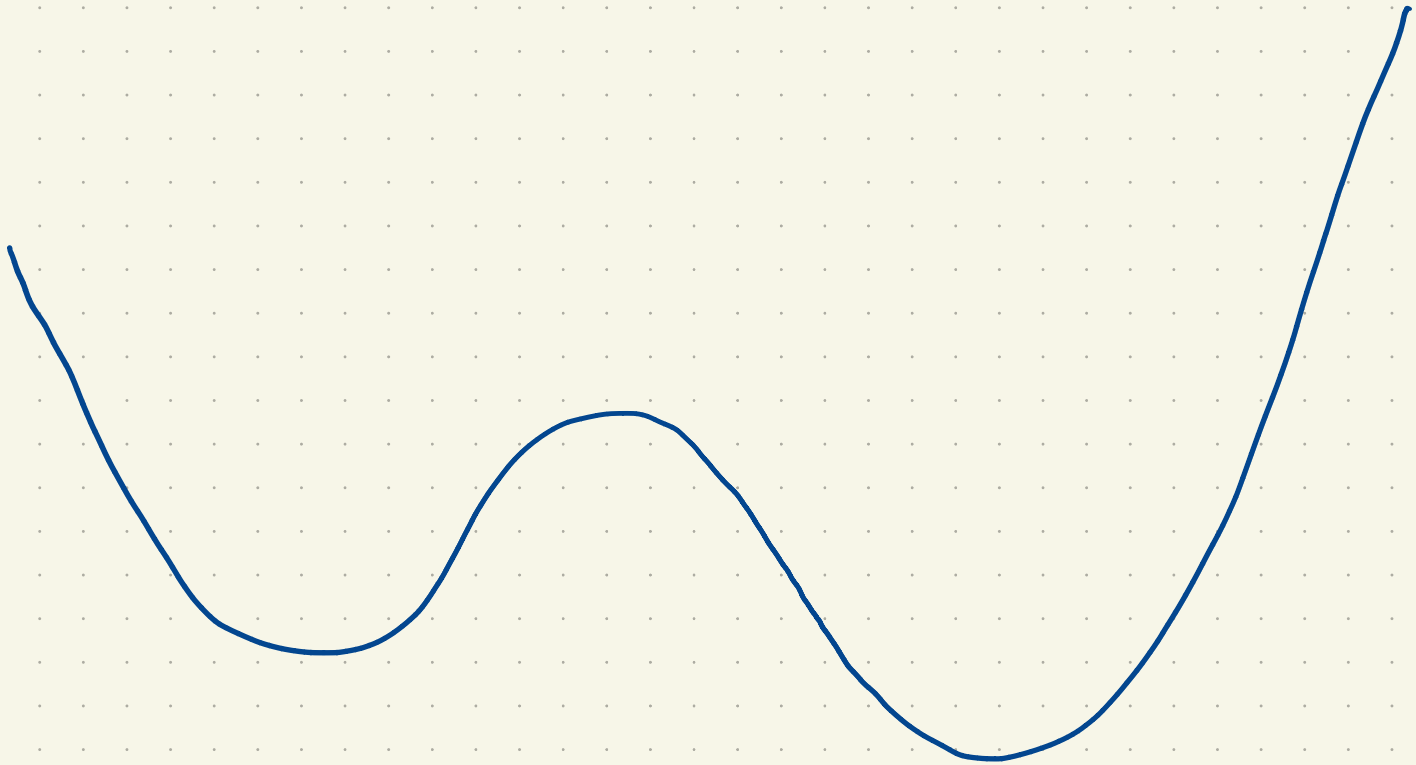
local   neither

$$f(x) = x^3$$

$$f'(x) = 3x^2 \geq 0$$

$$f'(0) = 0$$





Second Derivative Test

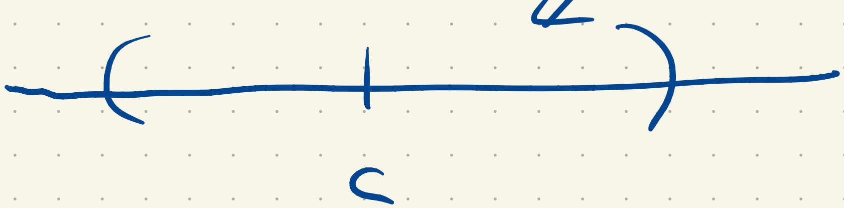
$f'(c) = 0$

local min?
local max?

$$f''(c) \quad \left[\begin{array}{l} > 0 \\ = 0 \\ < 0 \end{array}\right.$$

($f''(x)$ is continuous near c)

on this interval



$f''(x) > 0$

what does this say about $f'(x)$

$g(x) = f'(x)$

$g(x) = f'(x)$

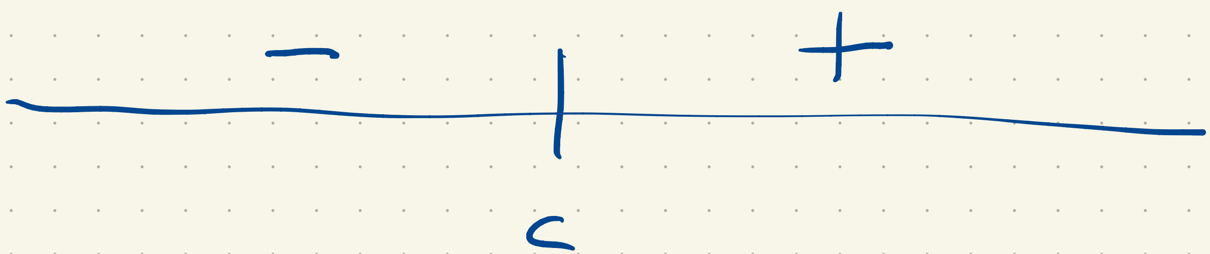
$g'(x) > 0$ on this interval

$f''(x) > 0$

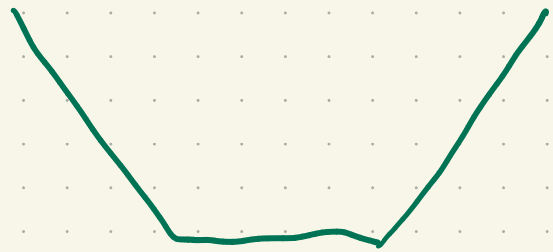
$g(x)$ is (strictly) increasing

$f'(x)$ is strictly increasing

$f'(x)$



$f'(c) = 0$



local minimum

$$\begin{cases} f'(c) = 0 \\ f''(c) > 0 \end{cases} \quad (f''(x) \text{ continuous near } c)$$

$\Rightarrow$ local min at c

$$\begin{matrix} f'(c) = 0 \\ f''(c) < 0 \end{matrix} \Rightarrow \text{local max at } c$$

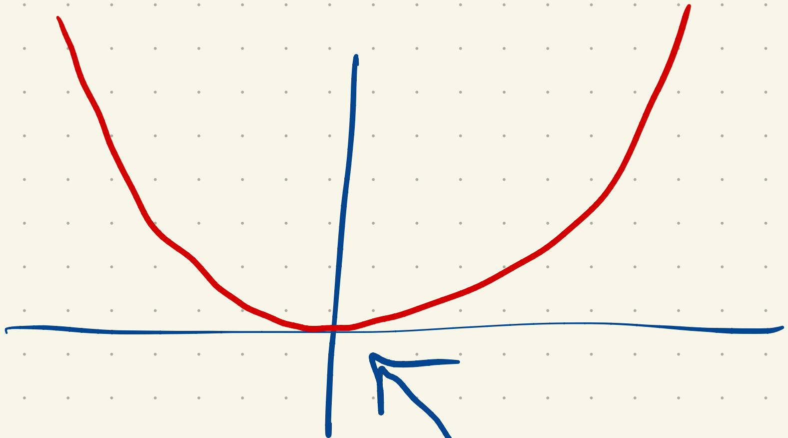
$f'(c) = 0$ want to know:

local min vs local max.

$f''(c) > 0 \Rightarrow$ local min

$f''(c) < 0 \Rightarrow$ local max

$f(x) = x^2$ $\quad f'(x) = 2x$ $\quad f''(x) = 2$



$f'(0) = 2 \cdot 0 = 0$

$f''(0) = 2 > 0$

local min.

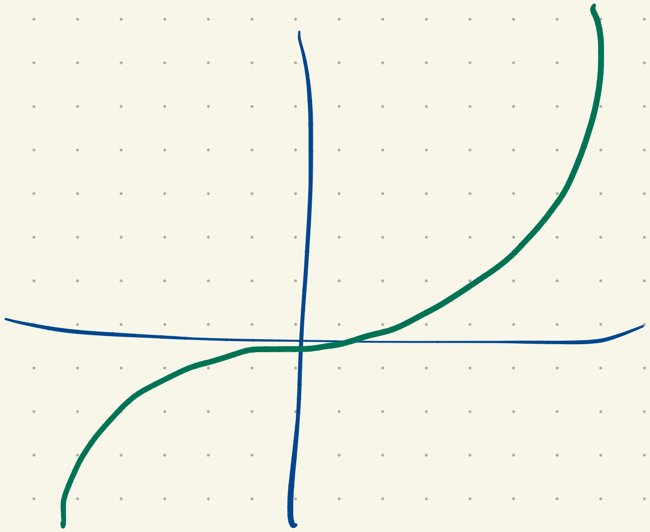
$f'(c) = 0$

$f''(c) = 0$

$\rightarrow$ inconclusive!

$f(x) = x^3$ $\quad f'(x) = 3x^2$ $\quad f''(x) = 6x$

$f'(0) = 0$ $\quad f''(0) = 0$



$f(x) = x^2$

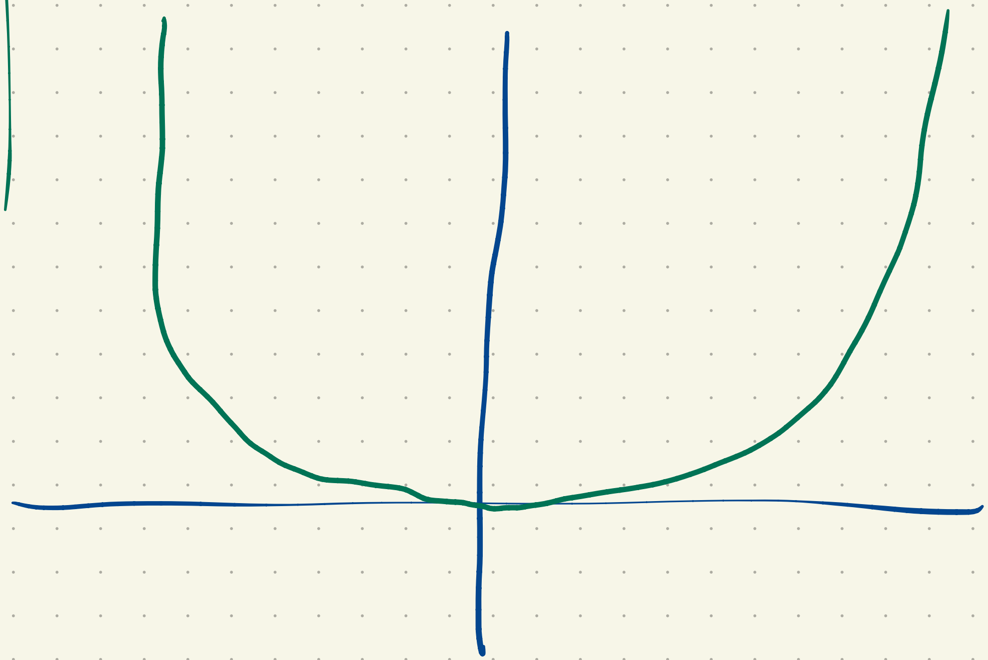
$f'(x) = 2x$

$f''(x) = 2$

$f'(0) = 0$

$f''(0) = 2$

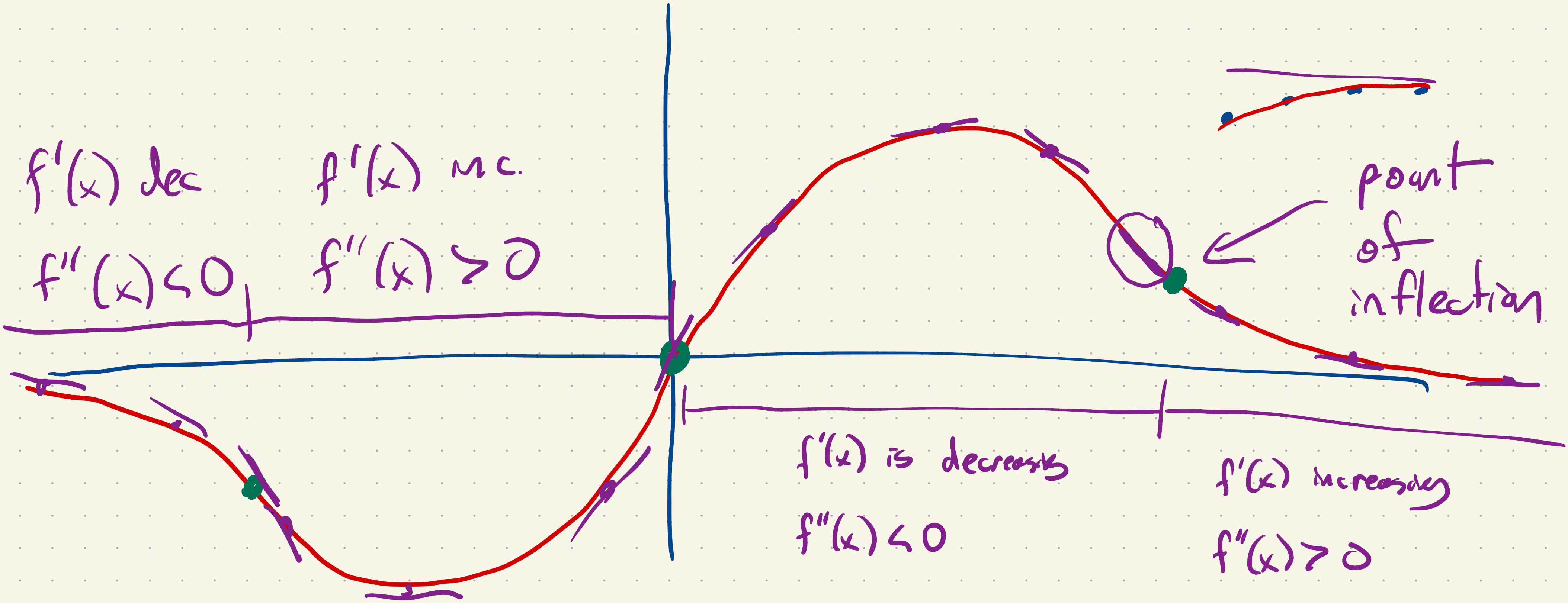
$f(x) = x^4$



$f(x) = -x^4$

$f'(x) = 4x^3$ $f'(0) = 0$

$f''(x) = 12x^2$ $f''(0) = 0$

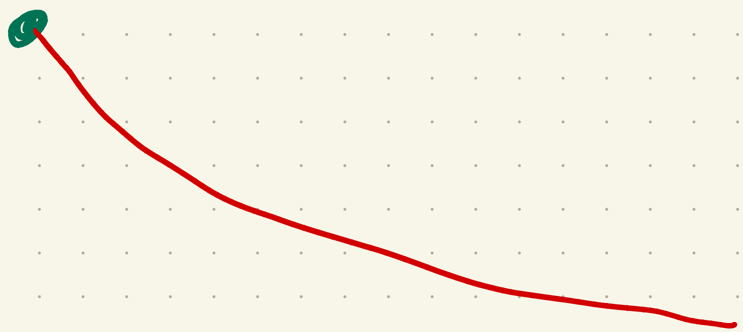

f'(x) dec
f''(x) < 0
f'(x) inc.
f''(x) > 0
point of inflection
f'(x) is decreasing
f''(x) < 0
f'(x) increasing
f''(x) > 0

$$f(x) = \frac{x}{1+x^2}$$

$\underbrace{f''(x) < 0}$

↳ downward U

concave down



$\underbrace{f''(x) > 0}$

↳ upword U

concave up

$\cup$ $\cap$

$f''(c) = 0$ ← point of inflection

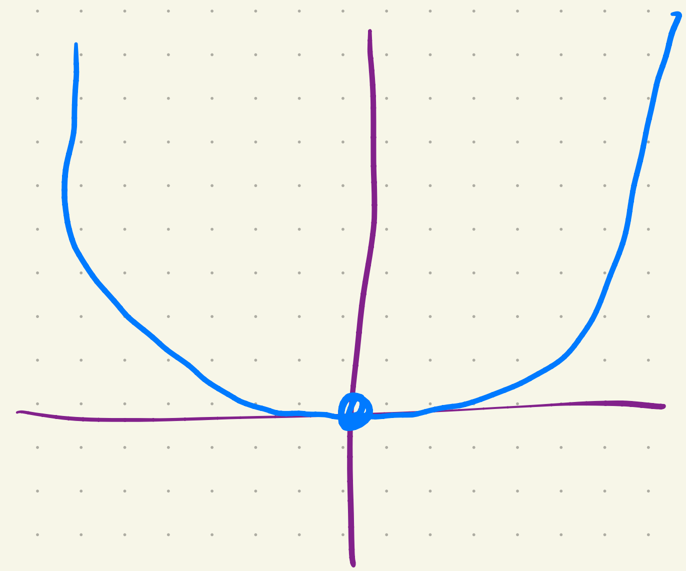
so long as $f''(x)$ changes sign

at c.

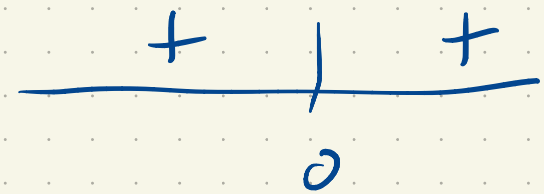
$f(x) = x^4$ $f(0) = 0$

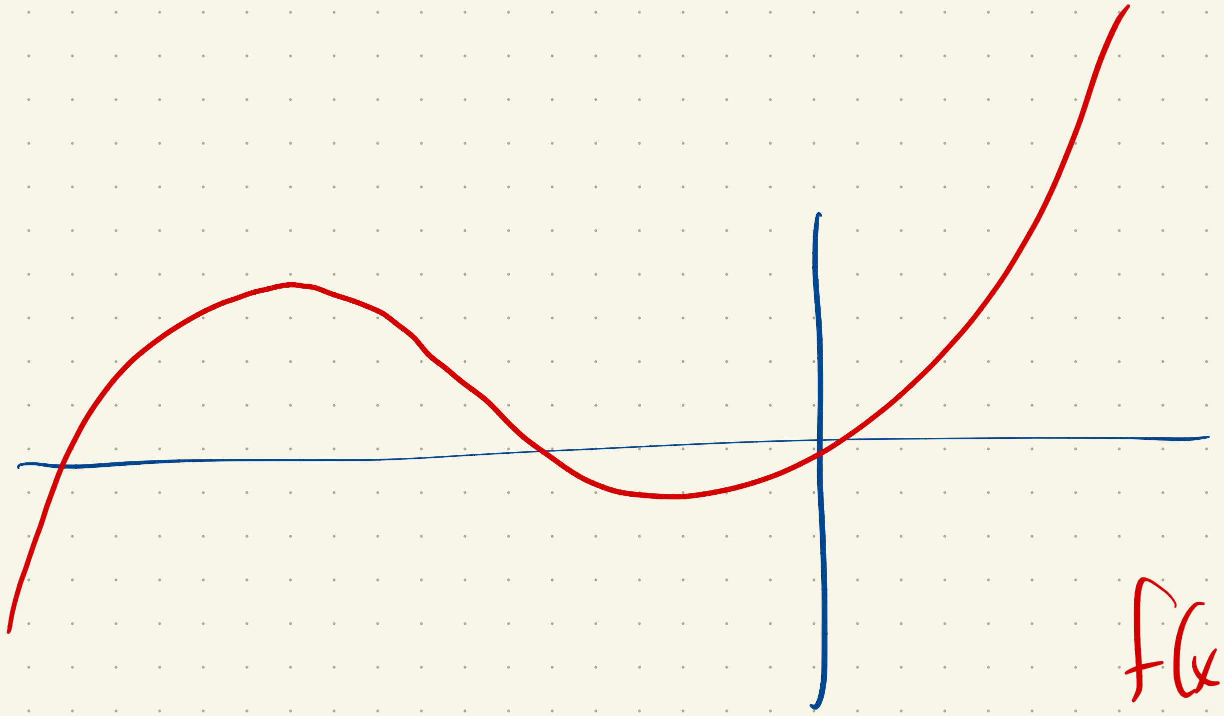
$f'(x) = 4x^3$ $f'(0) = 0$

$f''(x) = 12x^2$ $f''(0) = 0$

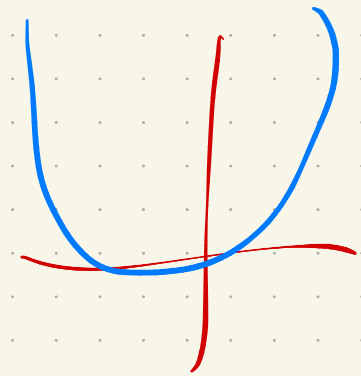


$f''(x)$





$f(x) = x^2$



$f''(x) = 2$