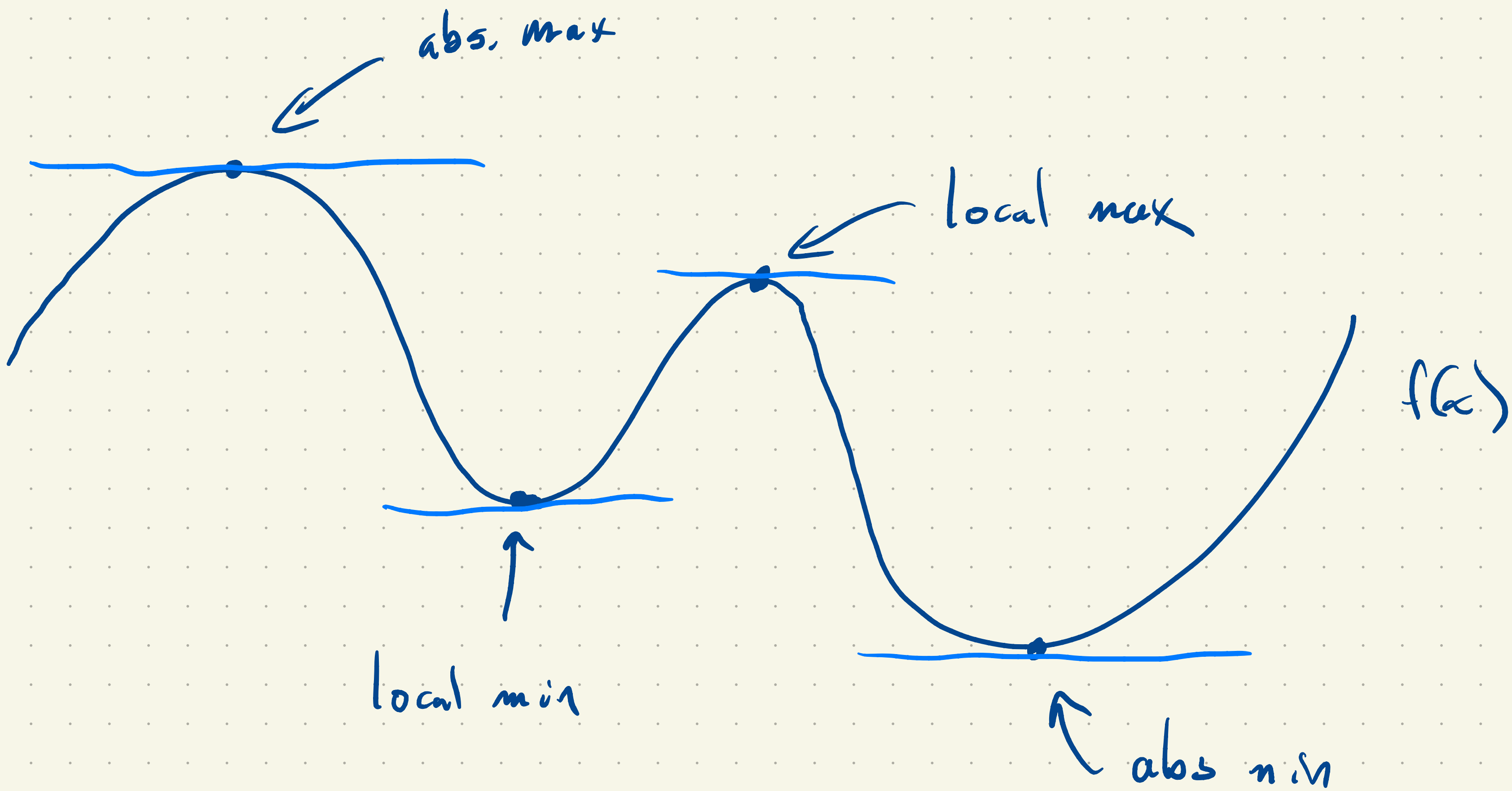
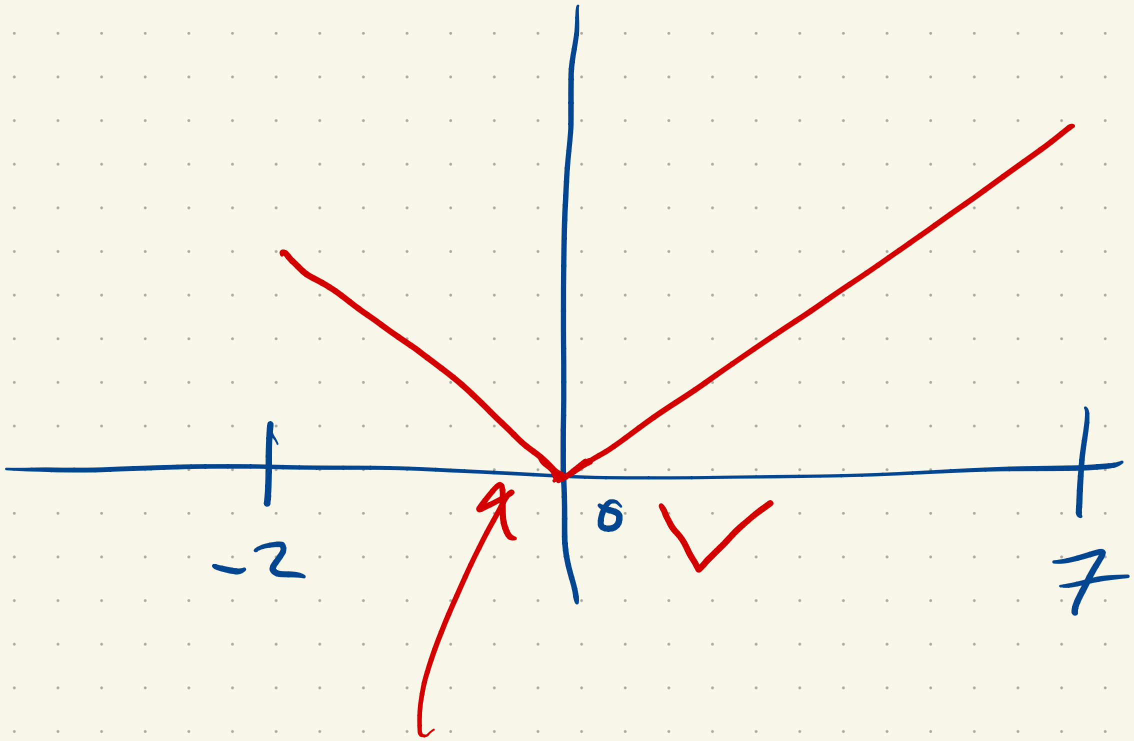
Optimization



At these points $f'(x) = 0$



$f'(x)$ DNE

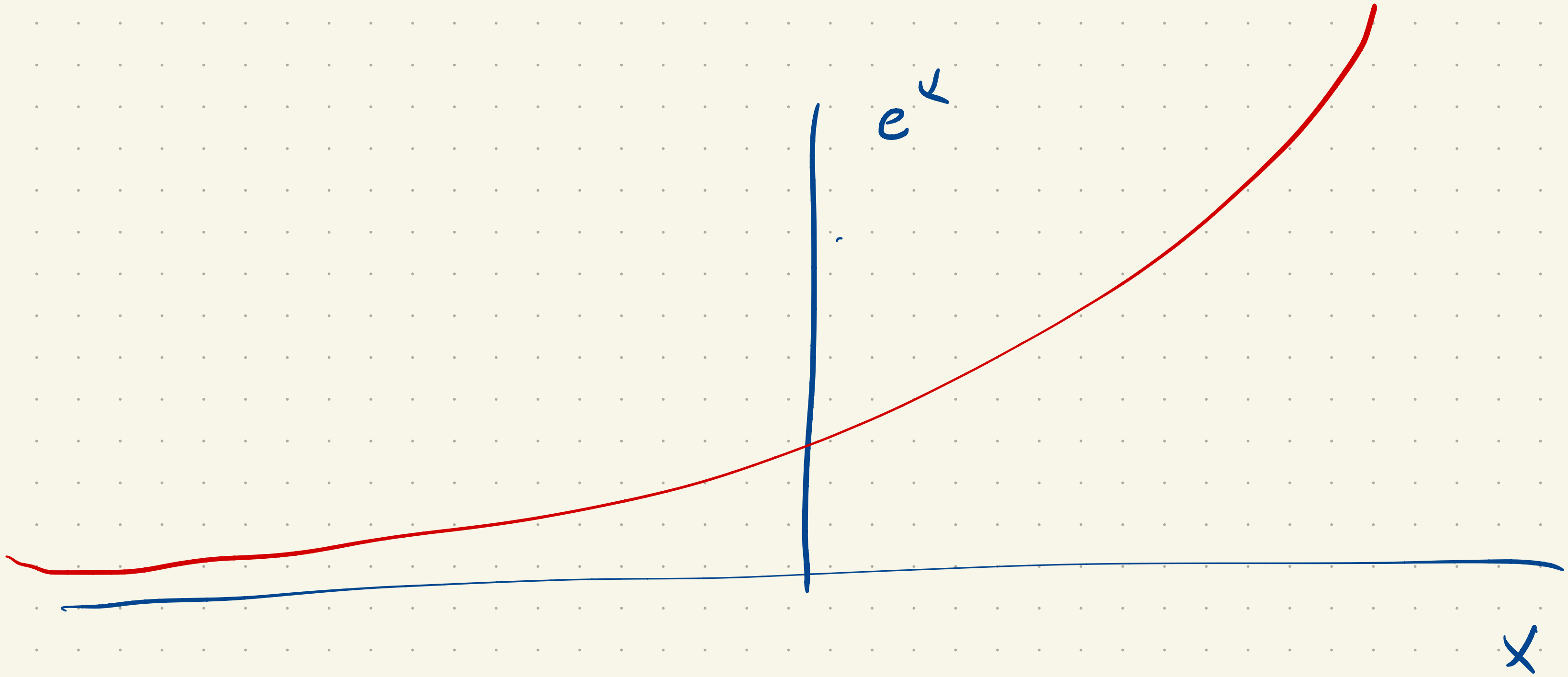
$f'(c) = 0$

or

$f'(c)$ DNE

critical number

$c \leftarrow$ point



This has neither an abs. max nor an abs. min.

- $f(x)$, continuous
- domain: closed, bounded interval

  $[a, b]$ $\qquad$ $(-\infty, \infty)$ $\qquad$ $(0, \infty)$

Such functions are guaranteed to have

an absolute min / max.

These will occur at one of

a) a critical number

b) an endpoint

e.g. $f(x) = xe^{-x}$ on $[0, 3]$

$$f'(x) = 1 \cdot e^{-x} + x \frac{d}{dx} e^{-x}$$

$$= e^{-x} + x(-1)e^{-x}$$

$$= e^{-x}(1-x)$$

$f'(x)=0$

$f'(x)$ DNE

$$f'(1)=0$$

critical number

only one.

$$e^{-x}(1-x)=0$$

$$1-x=0$$

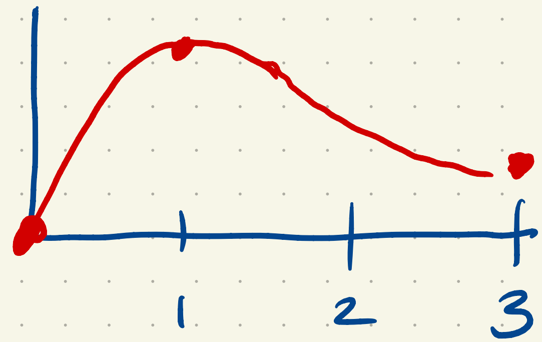
$$1=x$$

x	$f(x)$
0	0
1	$1 \cdot e^{-1} \approx 0.36$
3	$3 \cdot e^{-3} \approx 0.14$

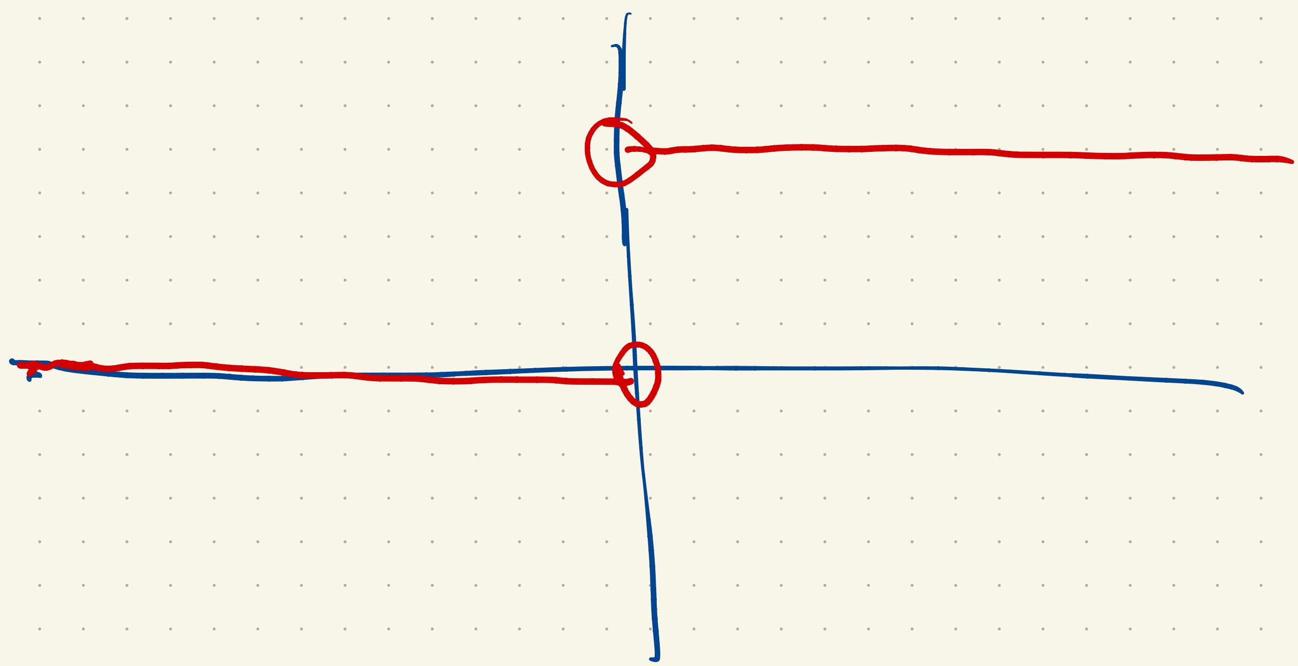
$f(x) = xe^{-x}$

abs min at $x = 0$

abs max at $x = 1$



$$\frac{d}{dx} 7 = 0$$

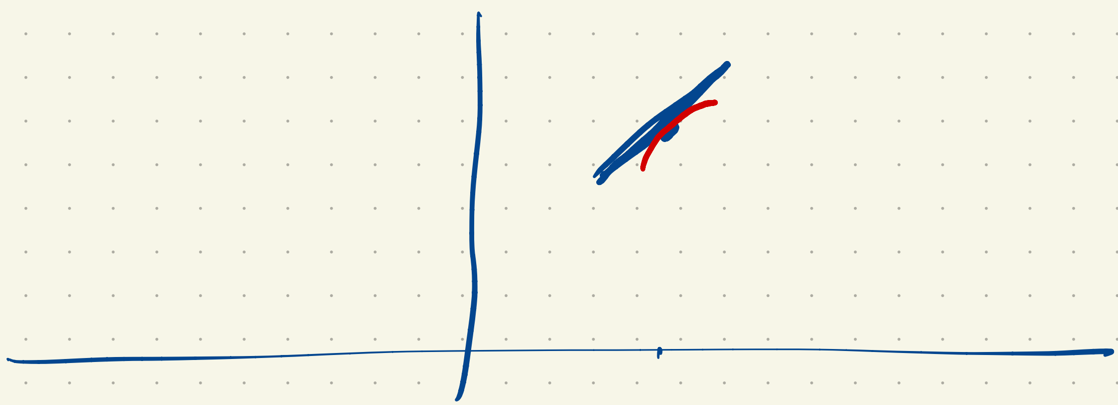


If a function $f(x)$ is defined on an interval and if $f'(x) = 0$ on its domain, then it is constant.

If $f(x)$ is defined on an interval and

if $f'(x) > 0$ at all points on the interval

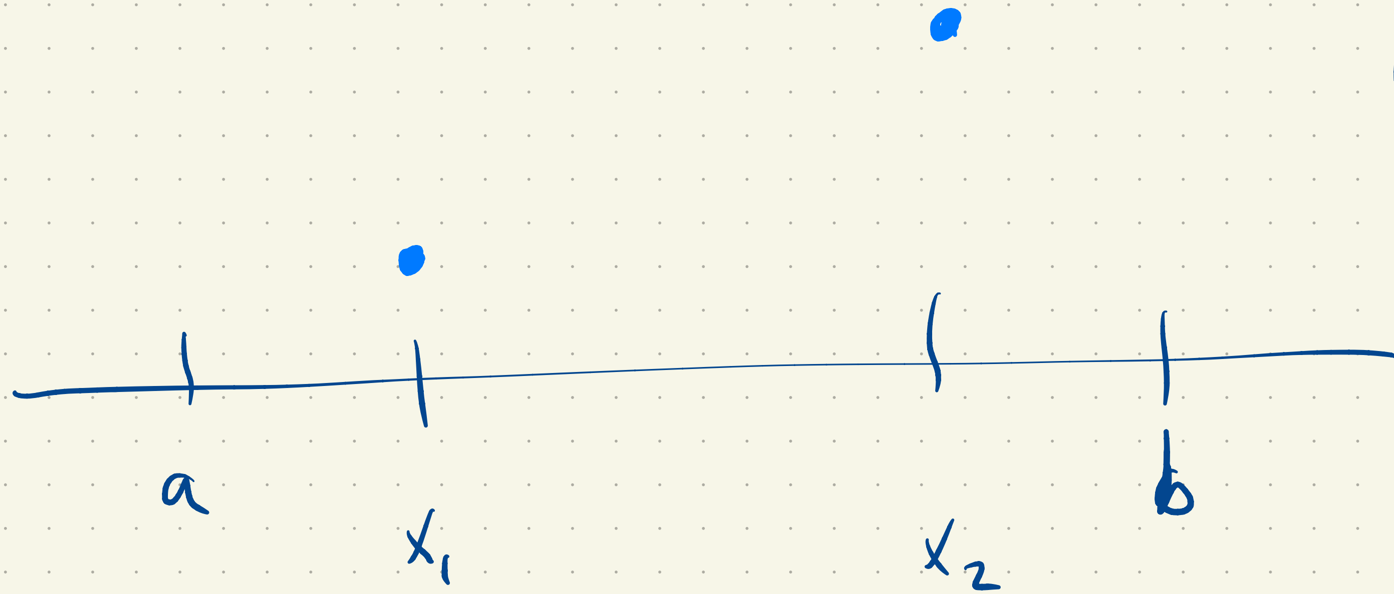
then



$\rightarrow$ the function is strictly increasing

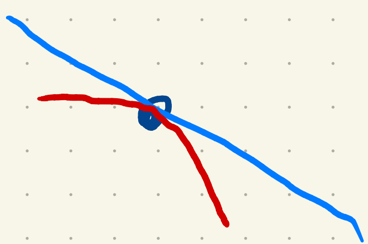
(if $x_1 < x_2$ then $f(x_1) < f(x_2)$)

(strictly increasing)



$f'(x) < 0$ on an interval?

↳ strictly decreasing



$x_1 < x_2$ implies

$f(x_1) > f(x_2)$

All these facts come from Mean Value Theorem.

$$f(x) = \frac{2}{3}x^3 + x^2 - 12x + 7$$

On what intervals is $f(x)$ increasing / decreasing?

increasing $\downarrow$ $f'(x) > 0$

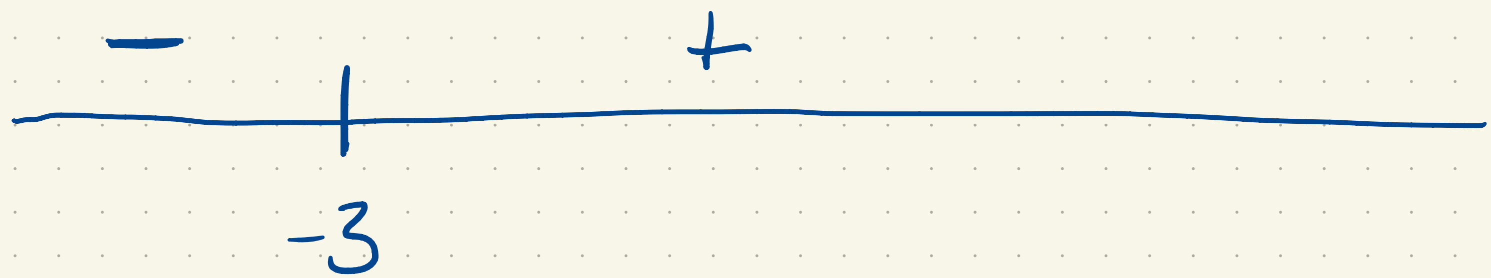
decreasing $\downarrow$ $f'(x) < 0$

$$f'(x) = 2x^2 + 2x - 12$$

$$= 2(x^2 + x - 6)$$

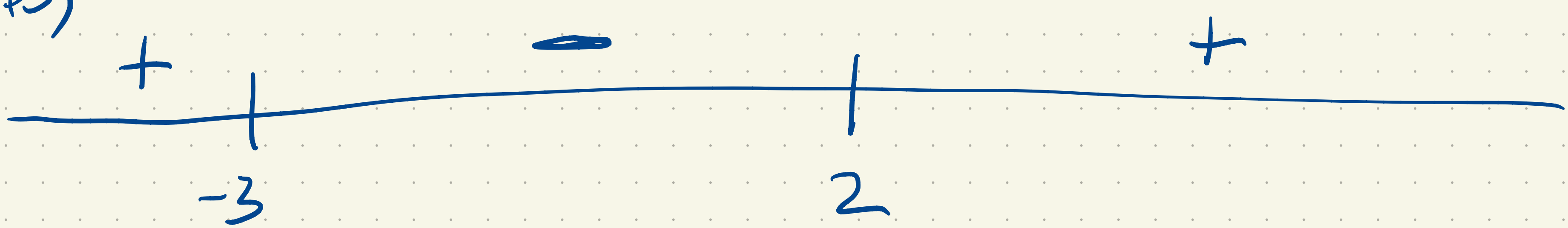
$$= 2(x+3)(x-2)$$

$x+3$



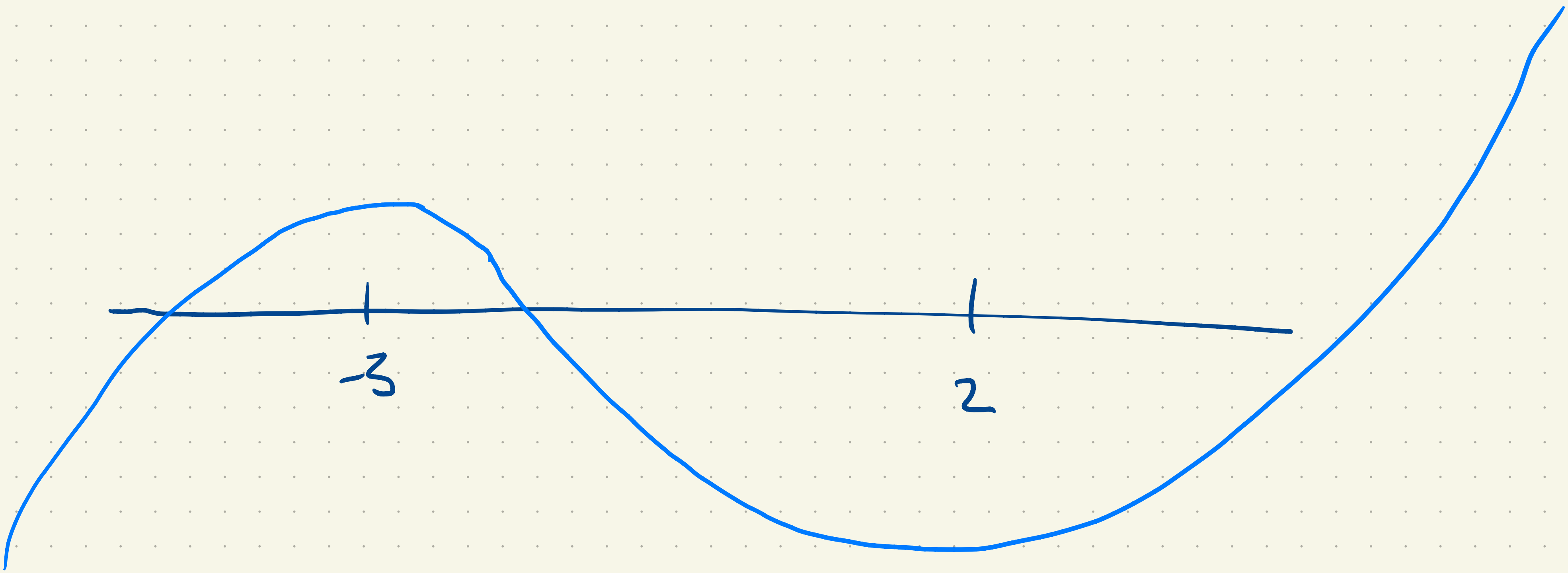
$x-2$

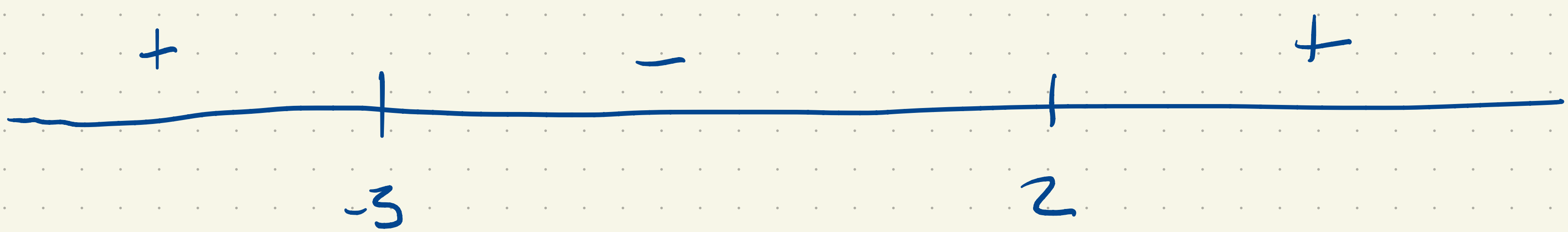
$(x-2)(x+3)$

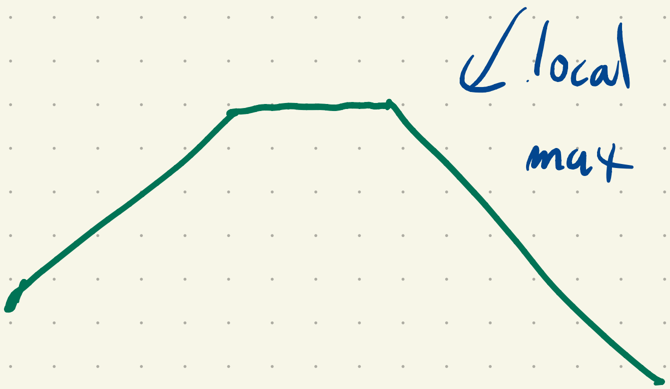


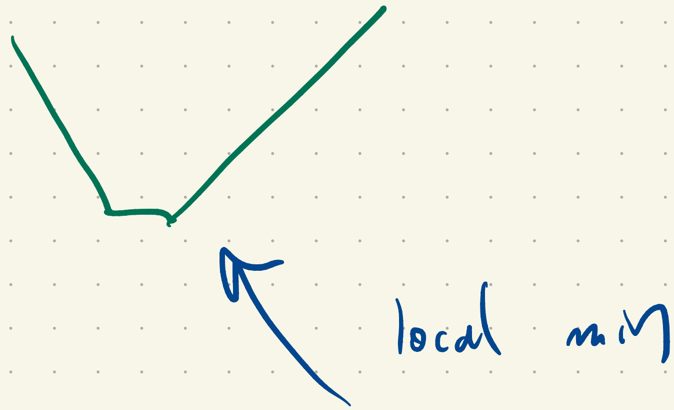
increasing on $(-\infty, -3)$ and on $(2, \infty)$

decreasing on $(-3, 2)$









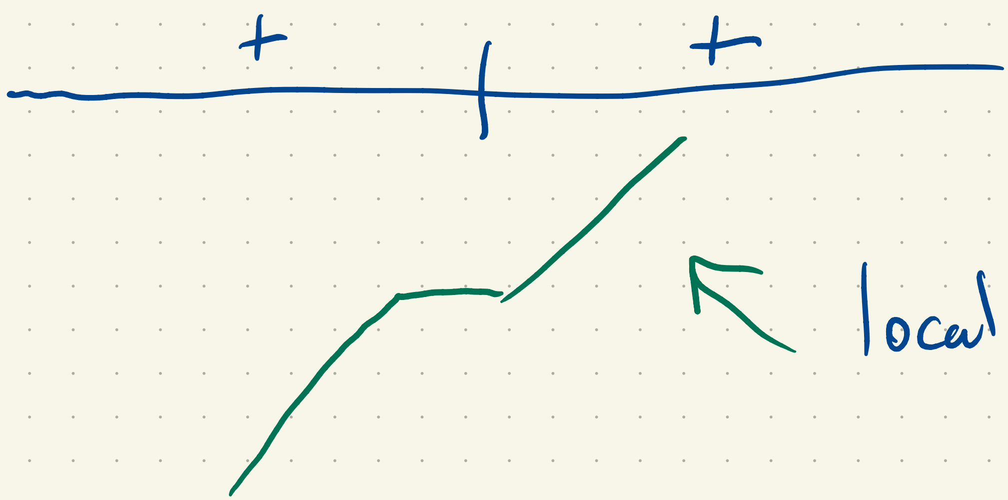
First Derivative Test

$f(x)$, defined on an interval around $x = c$

$f'(c) = 0$

If $f'(x)$ increases from neg to pos as x increases through $x = c$, $f(x)$ has a local min at $x = c$.

If $f'(x)$ decreases from positive to negative as x increases through $x = c$, $f(x)$ has a local max at $x = c$



local neither

If $f'(x)$ has the same sign on both sides of $x = c$ (both pos or both neg)

Then $f(x)$ has neither a local min nor a local max at $x=c$.

$f(x) = x^3$

$f'(x) = 3x^2 \qquad f'(0) = 0$

$+ \qquad\qquad + $

0