Optimization

looking for "the best"

↳ the biggest

the least

Suppose $f(x)$ is a function with

domain $\mathbb{R}$. If c is a point

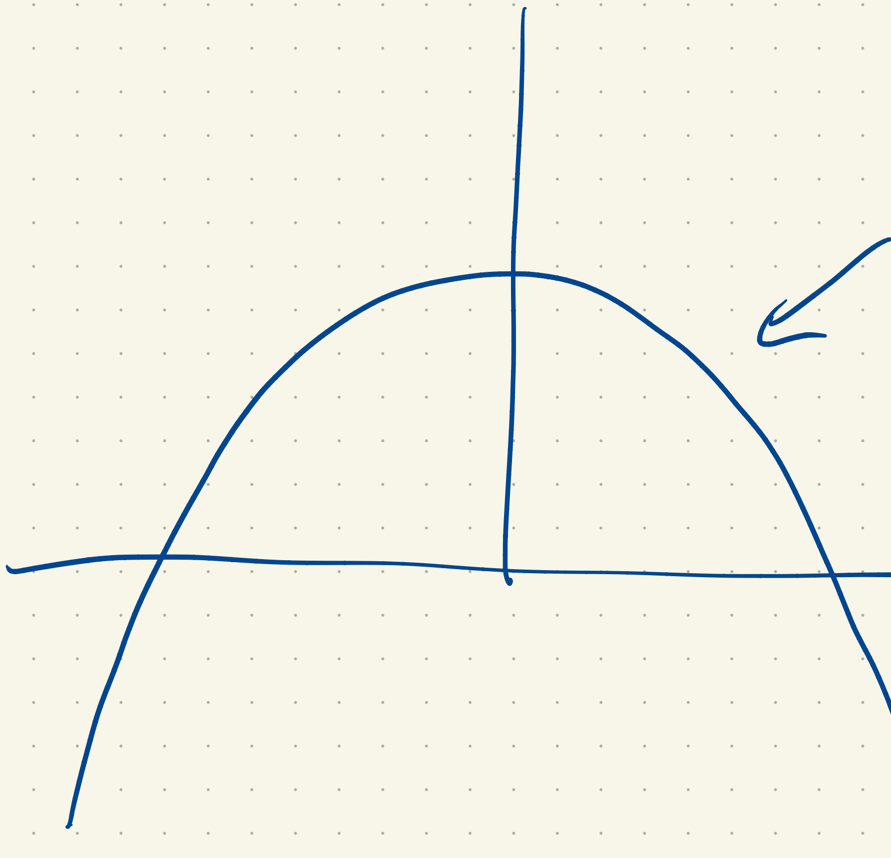
in $\mathbb{R}$ and if $f(c) \geq f(x)$ for all x in $\mathbb{R}$ we say $f(x)$ <u>attains</u>

<u>a</u> <u>maximum</u> at c and we call $f(c)$

the <u>maximum value</u> of $f(x)$

<u>absolute</u>

$f(x) = 1 - x^2$



f attains a maximum value at $x=0$

$f(0) = 1 \leftarrow$ the maximum value of $f(x)$.

Does this function have an absolute maximum value? $f(0) \geq f(x)$ for all x?

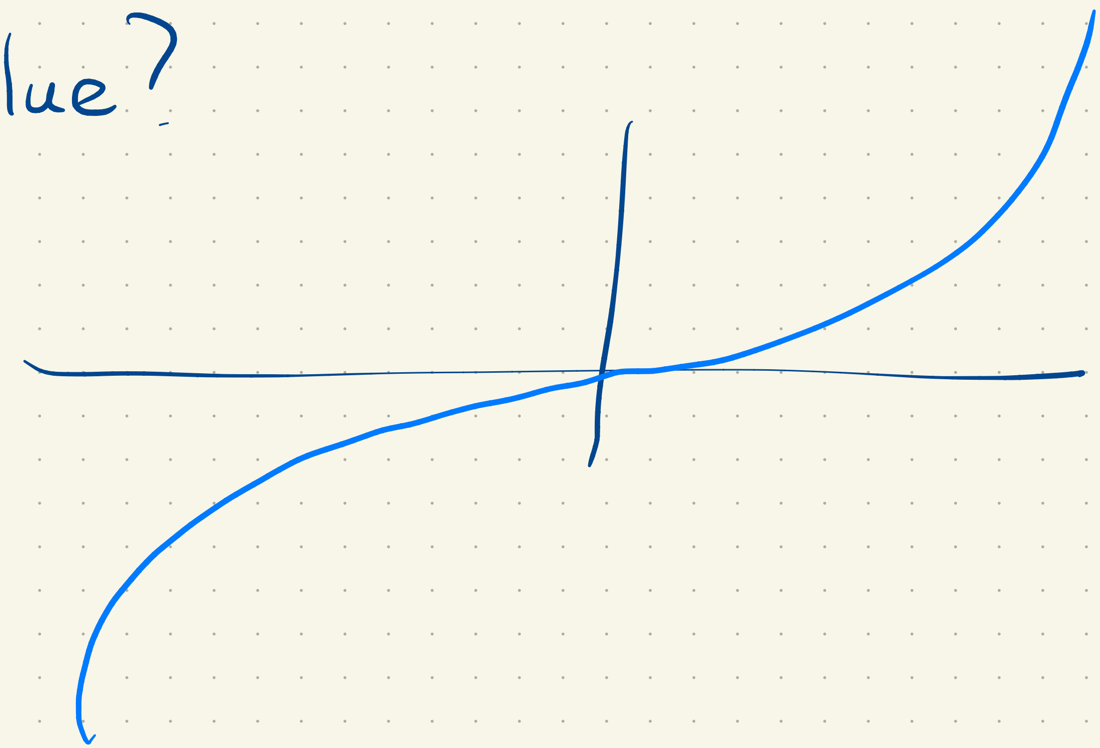
It attains its max value at every point.

For minimums, we say $f(x)$ attains an absolute (global) minimum at $x=c$ if $f(c) \leq f(x)$ for all x.

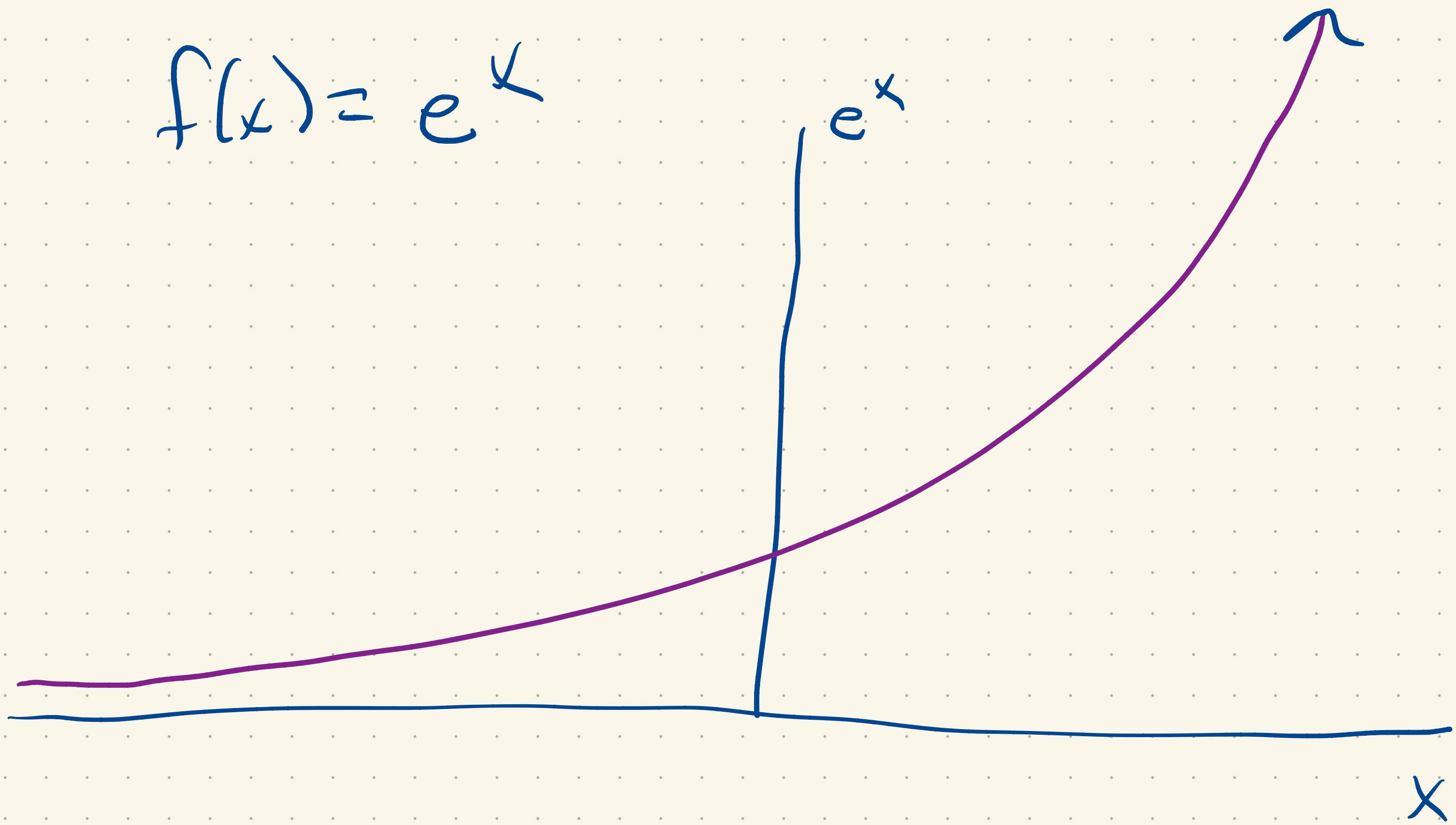
We call $f(c)$ the minimum value of $f(x)$.

Are there functions with domain $\mathbb{R}$ that do not attain either a minimum or a maximum value?

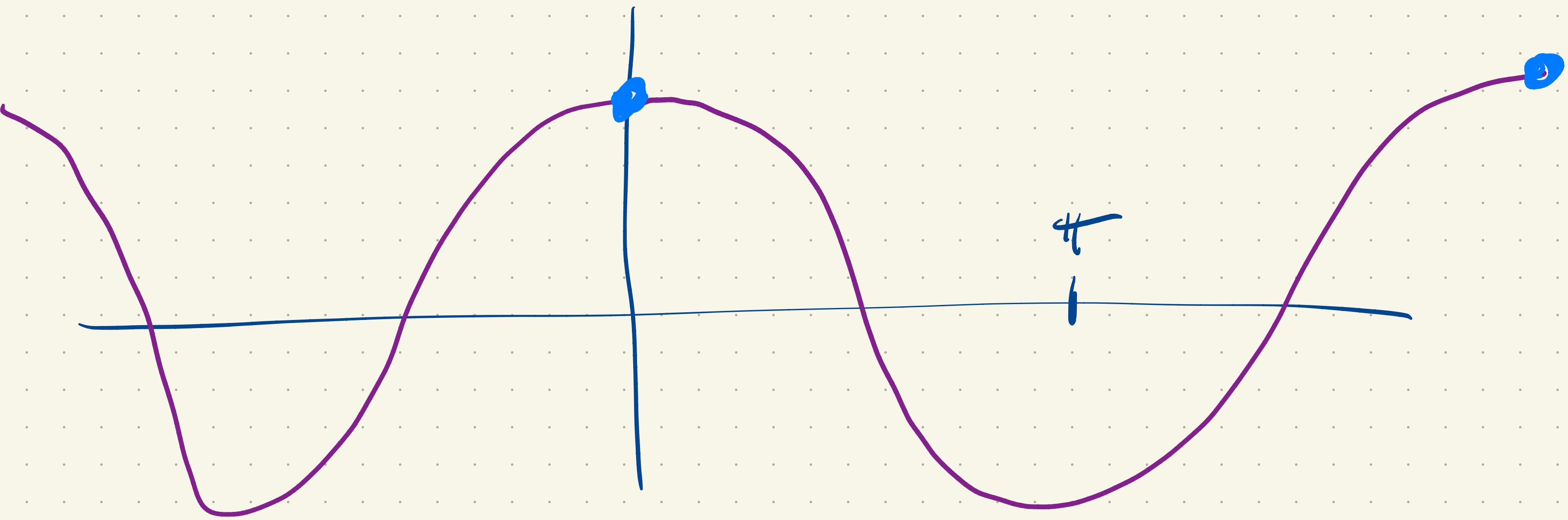
$$f(x) = x^3$$



$f(x) = e^x$



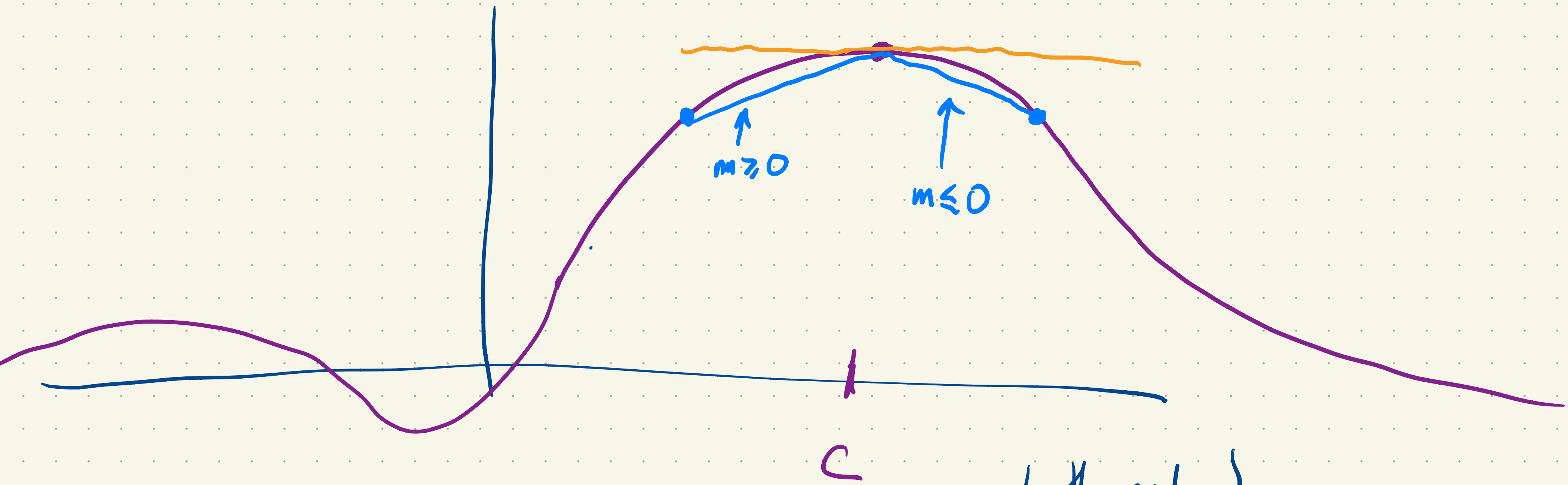
e^x does not acheive an absolute min or max.



$f(x) = \cos(x)$

This attains a abs. maximum value at $x = 0, 2\pi, 4\pi, 6\pi, -2\pi$

It attains an abs. minimum value at $x = \pi, 3\pi, 5\pi, -\pi, \ldots$

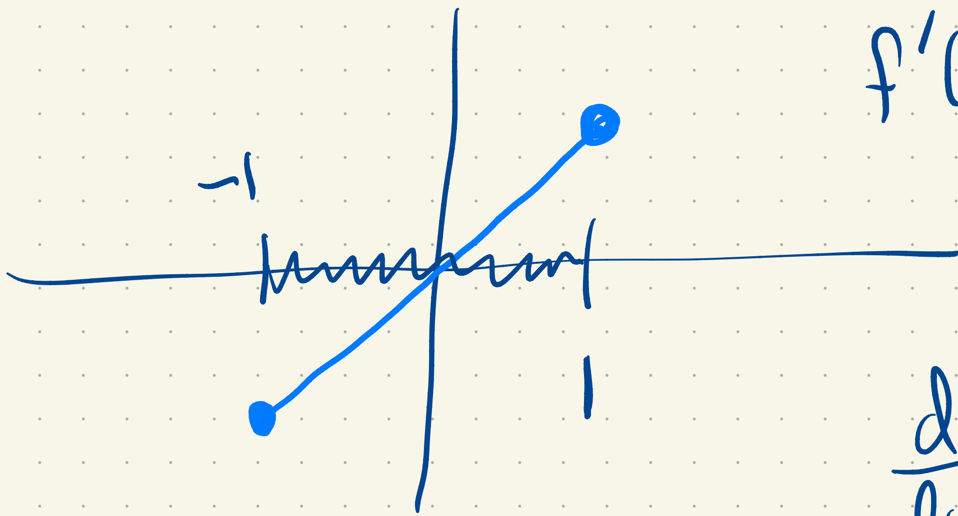


both sides!

Fact: If the domain of $f(x)$ contains an interval around c and if f attains an absolute maximum or minimum at $x = c$ and if $f'(c)$ exists then

$$f'(c) = 0$$

$f(x) = x$ on $[-1, 1]$



$f'(1) = 1$

$\frac{d}{dx} x = 1$