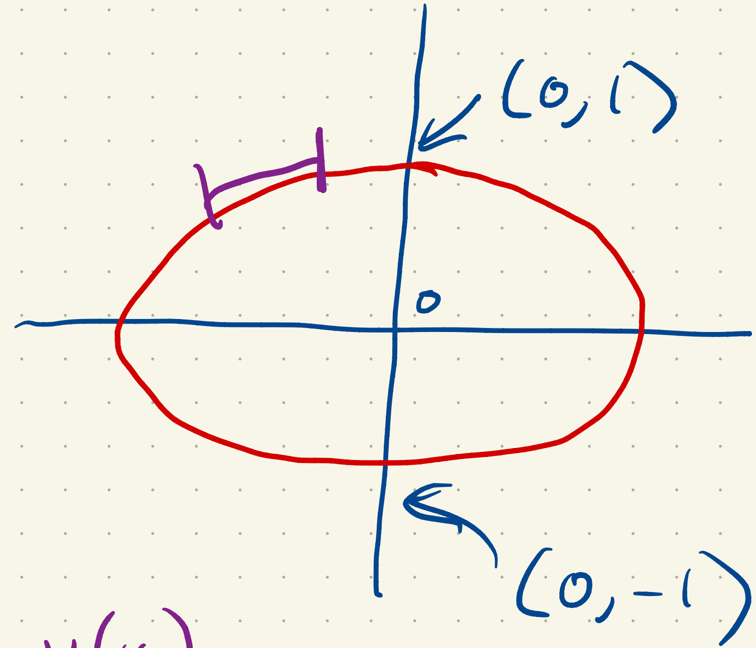
$$x^2 + 2y^2 = 2$$



1) Find $\frac{dy}{dx}$ on the curve.

$y(x)$

$$\frac{d}{dx}\left(x^2 + 2y^2\right) = \frac{d}{dx} 2$$

$$2x + 4y\frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = \frac{-2x}{4y} = -\frac{1}{2}\frac{x}{y}}$$

$(0,1)$ is on the curve

$0^2 + 2 \cdot 1^2 = 2$ ✓

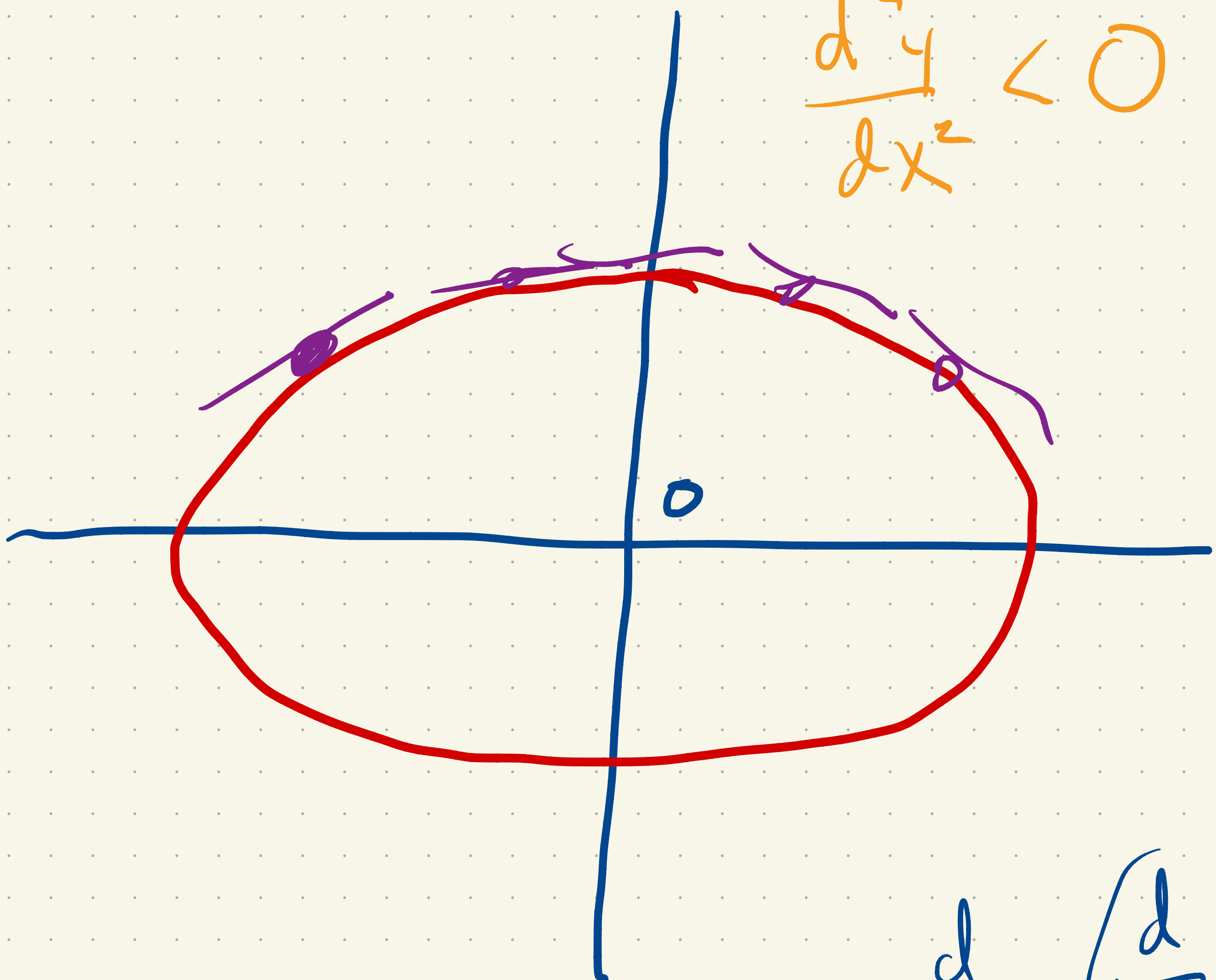
$\frac{dy}{dx}$ at $(0,1)$ is $-\frac{1}{2}\frac{0}{1} = 0$

x

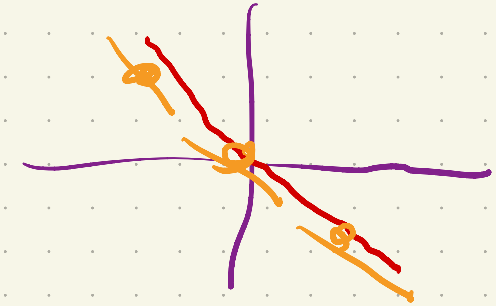
y

---

what is $\frac{d^2y}{dx^2}$ on the curve?



$$\frac{d^3y}{dx^2} < 0$$



$$\frac{dy}{dx}$$

$$\frac{d}{dx}\left(\frac{d}{dx} f(x)\right)$$

$$\left(\frac{d}{dx}\right)^2 f(x) = \frac{d^2}{(dx)^2} f(x)$$

$$y' = \frac{dy}{dx} = -\frac{1}{2}\frac{x}{y}$$

$$y\frac{dy}{dx} = -\frac{x}{2}$$

$$\frac{d}{dx}\left(y\frac{dy}{dx}\right) = \frac{d}{dx}\left(-\frac{x}{2}\right)$$

$$(y')^2 + yy'' = -\frac{1}{2}$$

$$\frac{d^2y}{dx^2} \qquad \frac{d}{dx}\frac{d}{dx}y$$

$$\frac{d}{dx}\, y(x)\, y'(x)$$

$$\parallel$$

$$\frac{dy}{dx}\cdot\frac{dy}{dx} + y\cdot\frac{d^2y}{dx^2}$$

$$y'\cdot y' + yy''$$

$$\left(\frac{d}{dx}\right)^2 = \frac{d^2}{dx^2}$$

$$y y'' = -\frac{1}{2} - (y')^2 \qquad y' = -\frac{1}{2}\frac{x}{y}$$

$$y'' = \frac{1}{y}\left[-\frac{1}{2} - (y')^2\right]$$

$$= \frac{1}{y}\left[-\frac{1}{2} - \left(-\frac{1}{2}\frac{x}{y}\right)^2\right]$$

$$= \frac{1}{y}\left[-\frac{1}{2} - \frac{1}{4}\frac{x^2}{y^2}\right]$$

$$= -\frac{1}{y}\left[\frac{1}{2} + \frac{1}{4}\frac{x^2}{y^2}\right]$$

$$x^2 + 2y^2 = 2$$

$$(x, y) = (0, 1)$$

$$y' = -\frac{1}{2}\frac{x}{y}$$

$$y' = -\frac{1}{2}\frac{0}{1} = 0$$

$$y'' = -\frac{1}{y}\left[\frac{1}{2} + \frac{1}{4}\frac{x^2}{y^2}\right]$$

$$y'' = -\frac{1}{1}\left[\frac{1}{2} + \frac{1}{4}\frac{0^2}{1^2}\right]$$

$$= -\frac{1}{2}$$

$$x^2 + 2y^2 = 2$$

want $\frac{d^2y}{dx^2}$, y''

y' $\frac{dy}{dx}$

$$2x + 4yy' = 0$$

$$y' = -\frac{1}{2}\frac{x}{y}$$

$$yy' = -\frac{1}{2}x$$

$$\frac{d}{dx}(yy') = \frac{d}{dx}\left(-\frac{1}{2}x\right)$$

$$y'y' + yy'' = -\frac{1}{2}$$

$$y'' = \left(-\frac{1}{2} - (y')^2\right)\frac{1}{y}$$

$$= -\frac{1}{y}\left[\frac{1}{2} + \left(\frac{1}{2}\frac{x}{y}\right)^2\right]$$

$$y'' = -\frac{1}{y}\left[\frac{1}{2} + \left(\frac{1}{2}\frac{x}{y}\right)^2\right]$$

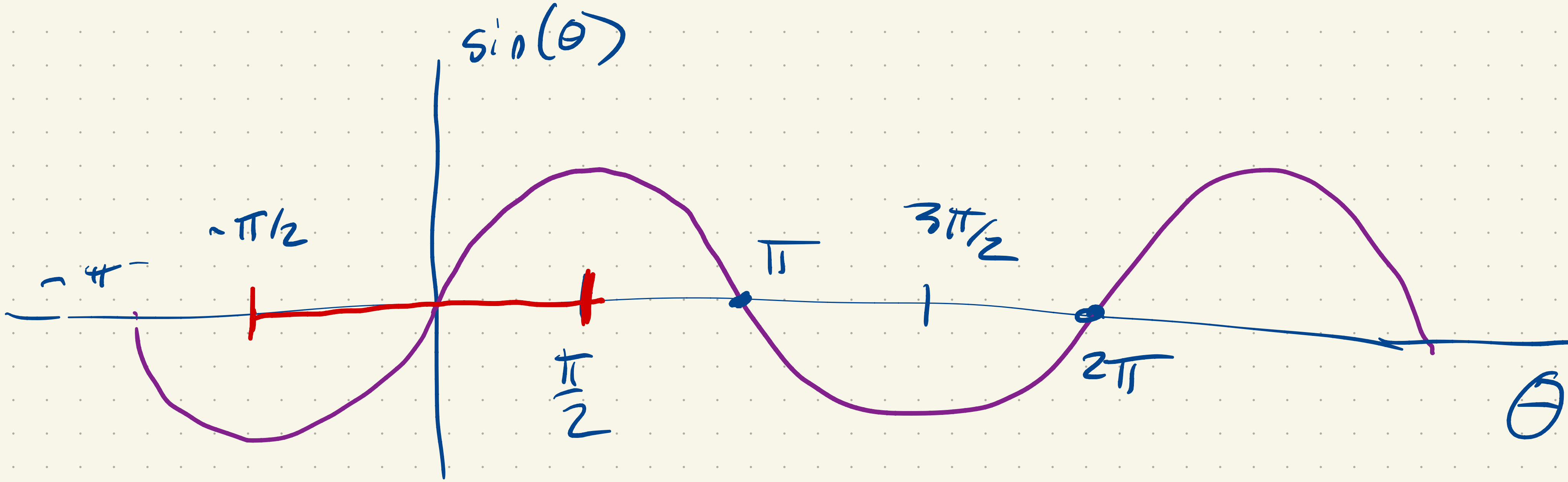
$$(y')^2 \qquad y' = -\frac{1}{2}\frac{x}{y}$$

$$\left(-\frac{1}{2}\frac{x}{y}\right)^2 = \left(\frac{1}{2}\frac{x}{y}\right)^2$$

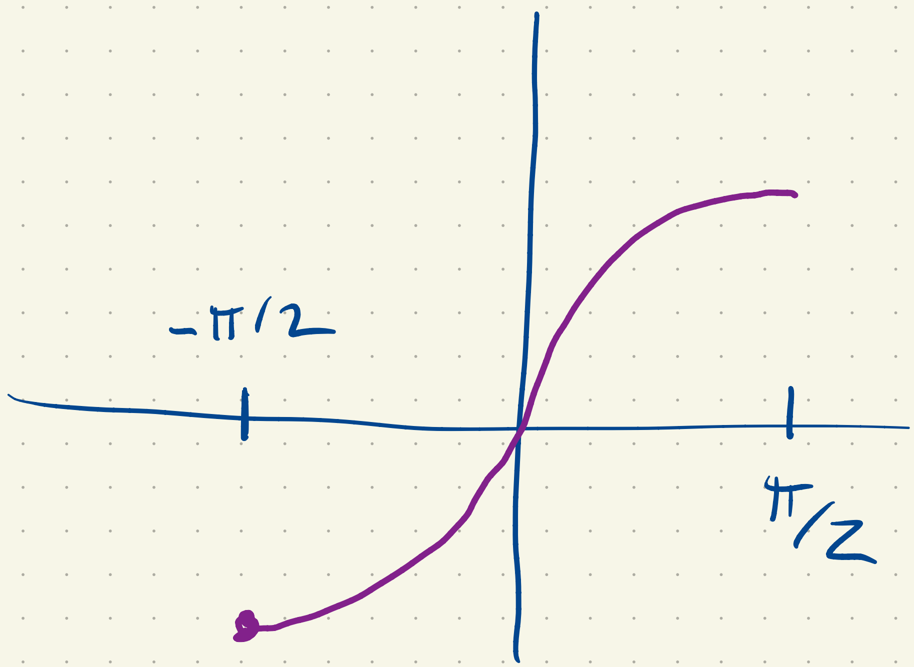
$$y'' = \left(-\frac{1}{2} - (y')^2\right)\frac{1}{y} = \frac{-1}{y}\left[\frac{1}{2} + (y')^2\right]$$

$$= \frac{-1}{y}\left[\frac{1}{2} + \left(-\frac{1}{2}\frac{x}{y}\right)^2\right]$$

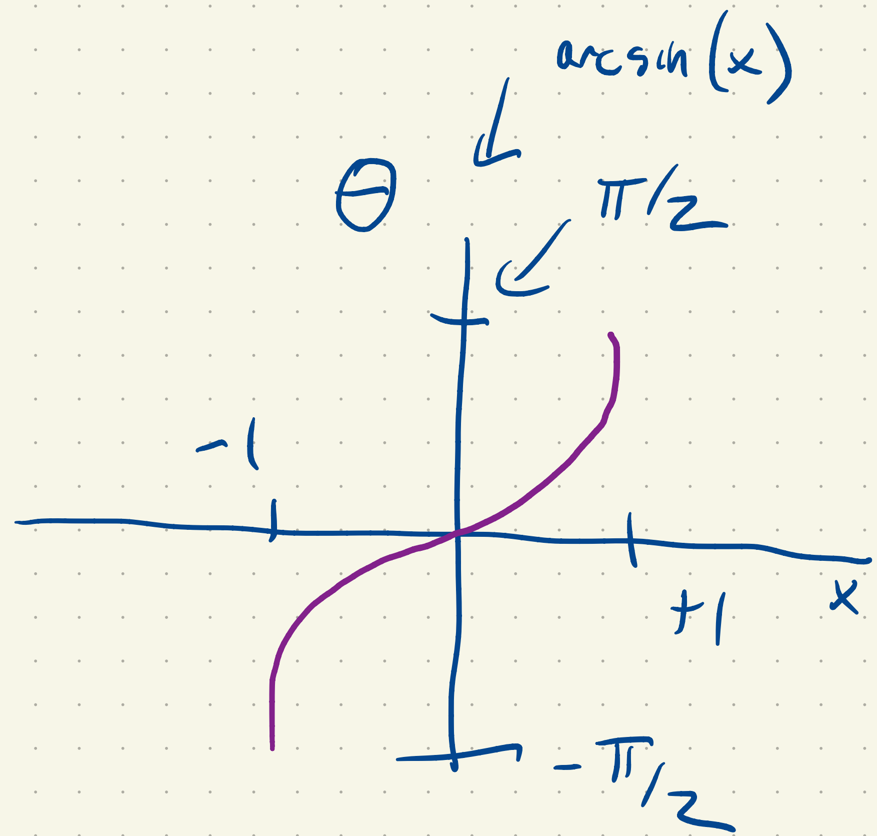
$$= \frac{-1}{y}\left[\frac{1}{2} + \left(\frac{1}{2}\frac{x}{y}\right)^2\right]$$



sin(θ)
-π
-π/2
π/2
π
3π/2
2π
θ



-π/2
π/2



arcsin(x)
θ
π/2
-1
+1
x
-π/2

$\arcsin(0) = 0 \rightarrow \sin(0) = 0$

$\arcsin(1) = \frac{\pi}{2} \rightarrow \sin\left(\frac{\pi}{2}\right) = 1$

$\arcsin(-1) = -\pi/2 \rightarrow \sin\left(-\frac{\pi}{2}\right) = -1$

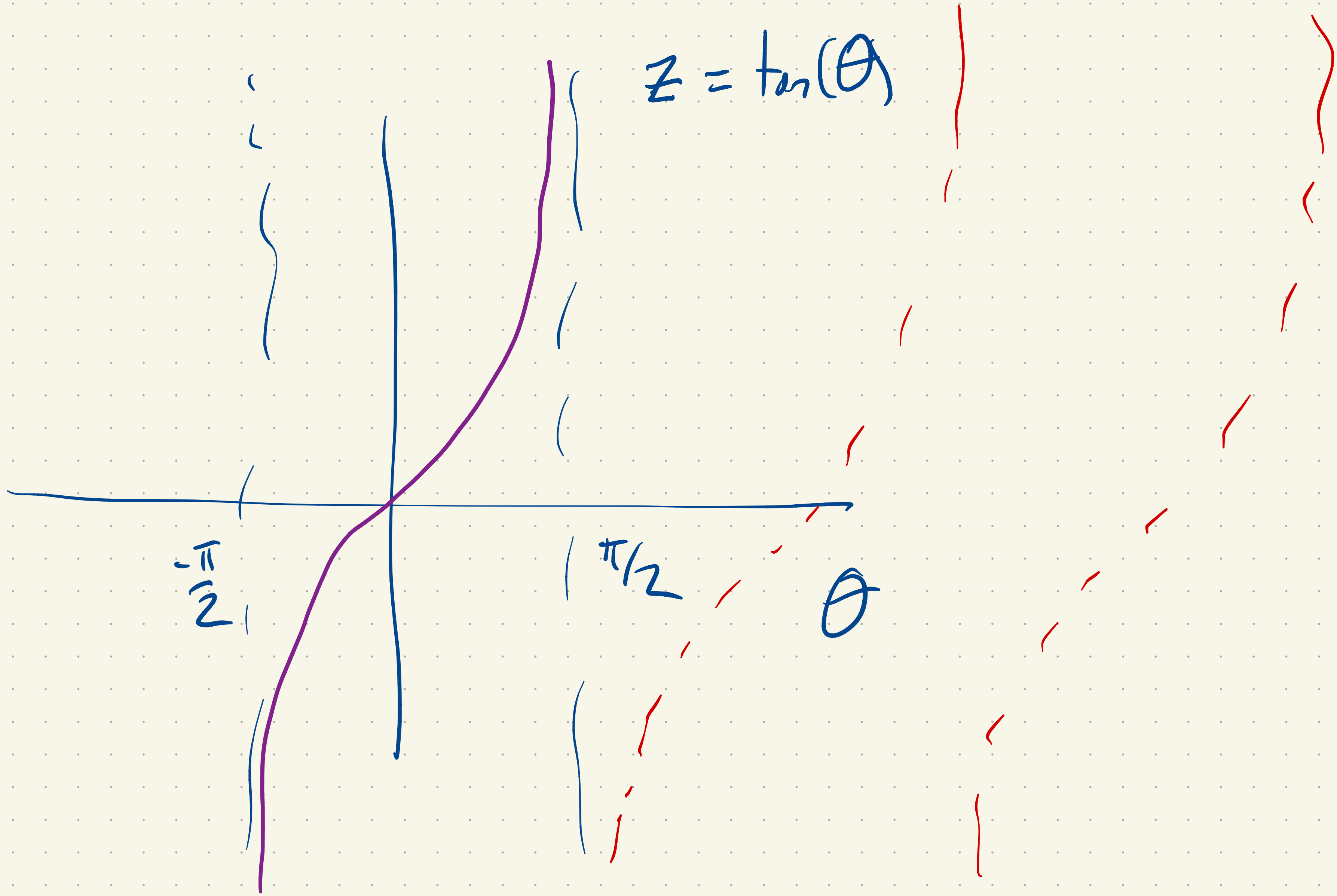
asin

$\sin^{-1}$

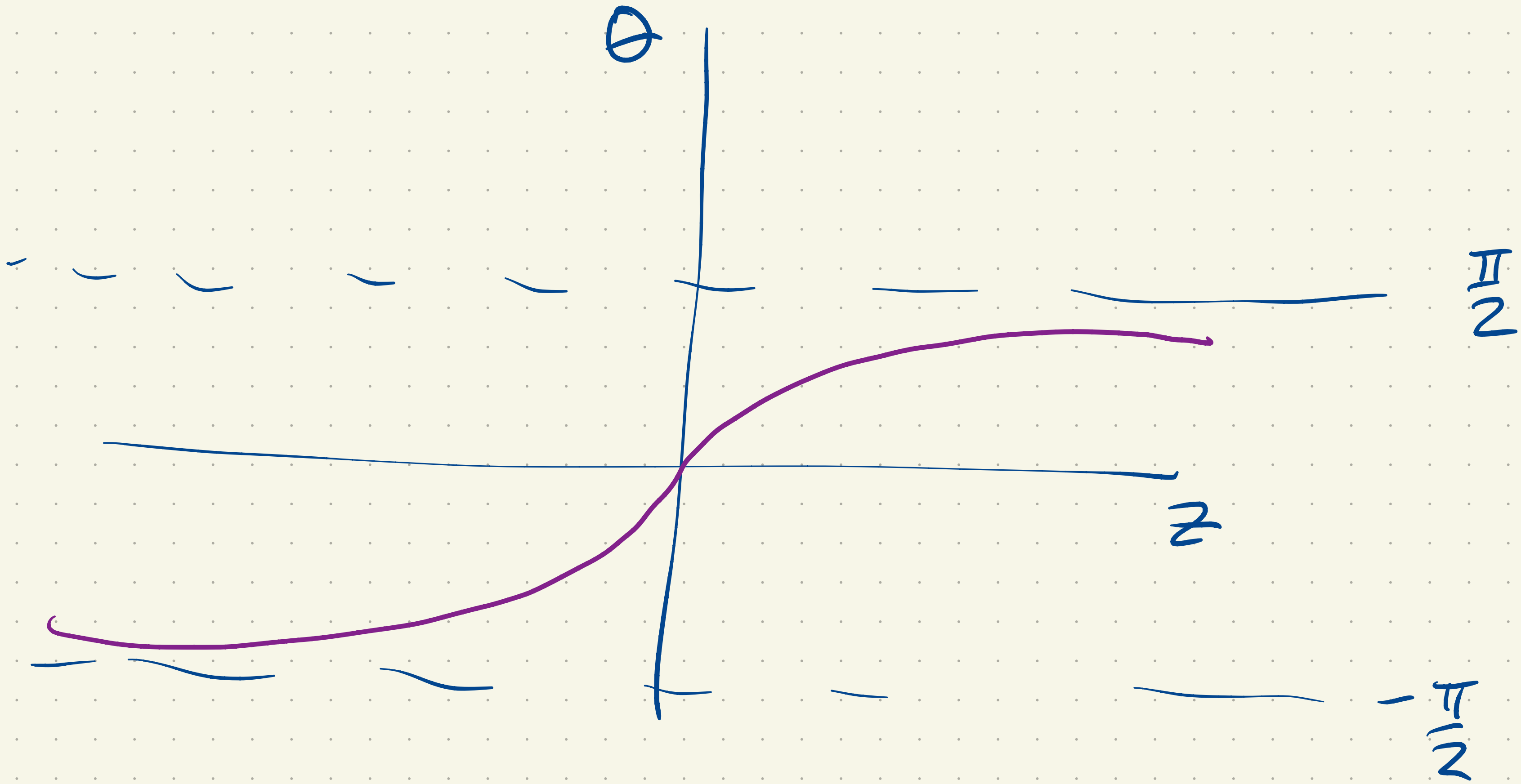
$\sin^2(x) = (\sin(x))^2$

$\sin^3(x) = (\sin(x))^3$

$\sin^{-1}(x) = \cancel{\frac{1}{\sin(x)}}$



$z = \tan(\theta)$
$-\frac{\pi}{2}$
$\pi/2$
θ



$\arctan(z)$

$\sin(\arcsin(z)) = z$

$\tan(\arctan(z)) = z$

$\arctan(\tan(\theta)) = \theta$

only true if $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

We want to compute $\frac{d}{dx} \arctan(x)$

$$\frac{d}{dx} \arcsin(x)$$

$$y = \arctan(x)$$

$$\tan(y) = \tan(\arctan(x))$$

$\tan(y) = x$ we want y'

$$\frac{d}{dx}\tan(y) = \frac{d}{dx}x$$

$$\frac{d}{dx}\arctan(x) = x$$

$$\sec^2(y)\cdot\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec^2(y)}$$

$$\sin^2(y)+\cos^2(y)=1$$

$$\frac{\sin^2(y)}{\cos^2(y)}+1 = \frac{1}{\cos^2(y)}$$

$$y' = \frac{1}{\sec^2(y)}$$

$$\tan^2(y)+1 = \sec^2(y)$$

$$= \frac{1}{1+\tan^2(y)}$$

$$y' = \frac{1}{1+x^2}$$

$$\boxed{\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}}$$

$$y = \arctan(x)$$

$$\tan(y) = x$$

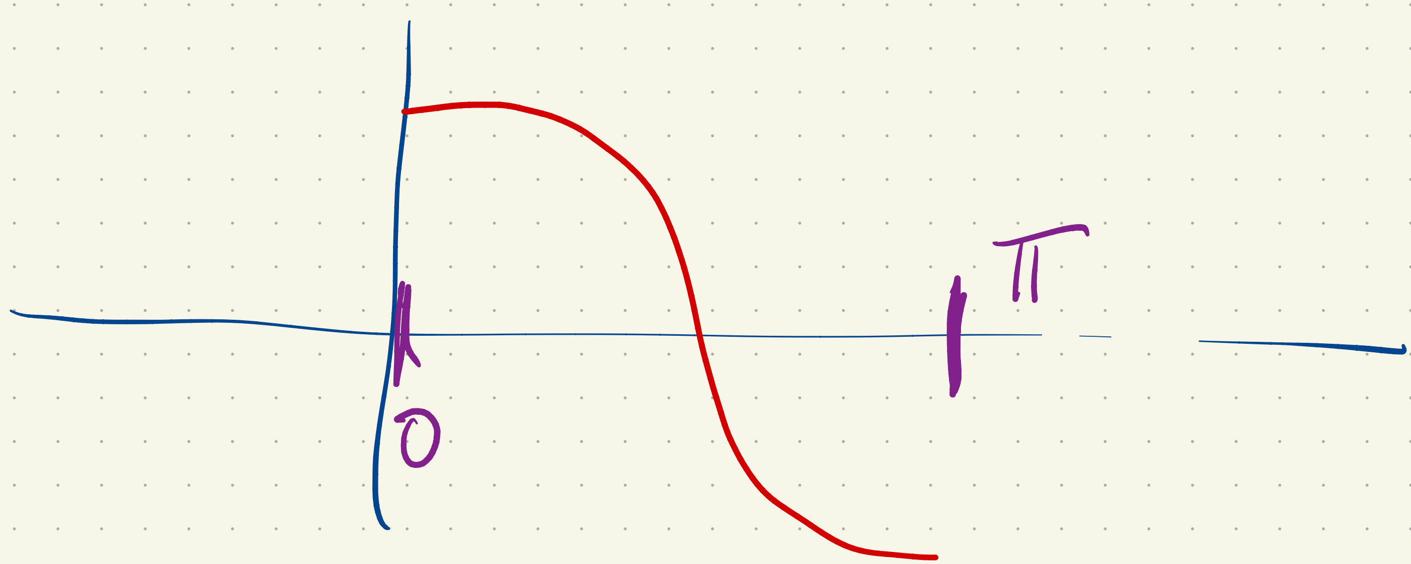
$$\frac{d}{dx} \tan(y) = 1$$

$$\sec^2(y) \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec^2(y)} = \frac{1}{1+\tan^2(y)} = \frac{1}{1+x^2}$$

$$\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \arccos(x) = \frac{-1}{\sqrt{1-x^2}}.$$



$$\frac{d}{dx} \operatorname{arcsec}(x)$$

$$\left[\frac{d}{dy}\sec(y)\right] y'$$

$$y = \operatorname{arcsec}(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sec(y) = x$$

$$\tan^2(y) = \sec^2(y) - 1$$

$$\underbrace{\sec(y)}\tan(y) \cdot y' = 1$$

$$\tan(y) = \pm\sqrt{\sec^2(y) - 1}$$

$$\pm x\sqrt{x^2 - 1} \cdot y' = 1$$

$$= \pm\sqrt{x^2 - 1}$$

$$y' = \frac{\pm 1}{x\sqrt{x^2 - 1}}$$