$$= \lim_{h \to 0} \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x)\left[\cos(h) - 1\right] + \sin(h)\cos(x)}{h}$$

$$= \sin(x) \lim_{h \to 0} \frac{\cos(h) - 1}{h} + \cos(x) \lim_{h \to 0} \frac{\sin(h)}{h}$$

$$= \sin(x) \cdot 0 + \cos(x) \cdot \underline{1}$$

$$= \cos(x)$$

$$\frac{d}{dx} \cos(x)$$

# Section 3.5 (Implicit Differentiation)

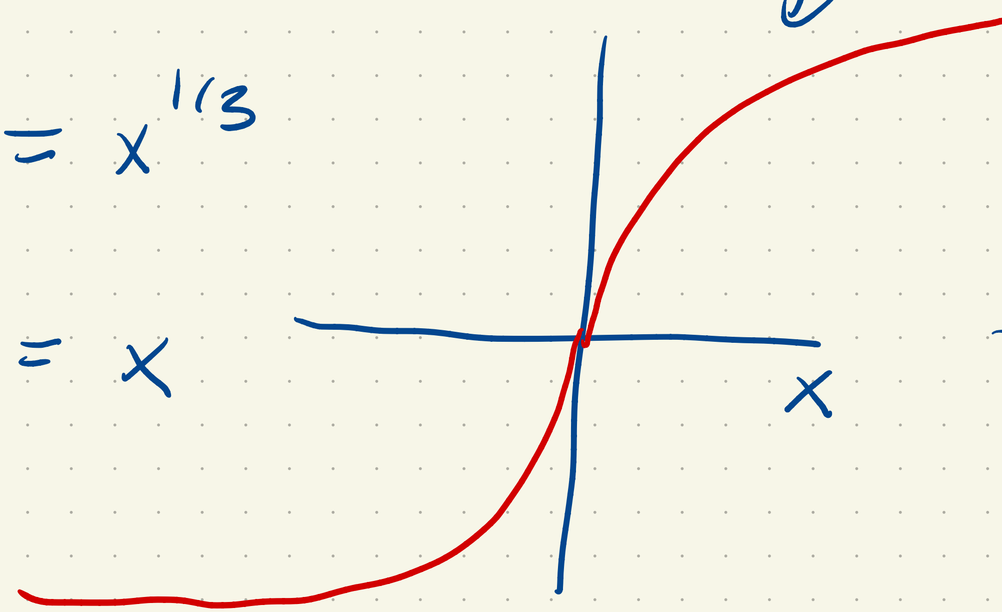
$$\frac{d}{dx}\ln(x)$$

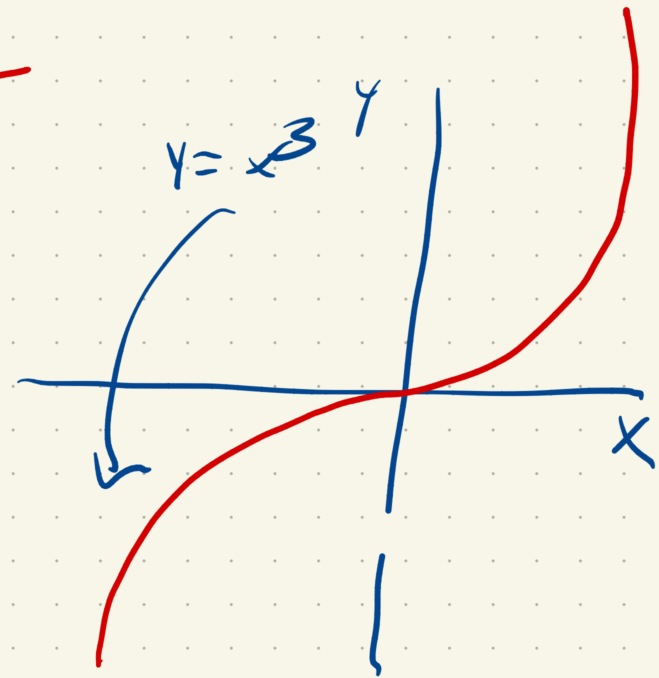
$$\frac{d}{dx}\arctan(x)$$

$$f(x) = x^{1/3} \longrightarrow \underbrace{\frac{1}{3}x^{-2/3}}_{f'(x)}$$

$$\frac{df}{dx}$$

$y = x^{1/3}$




$$y = x^{1/3}$$

$$y^3 = x$$

$\frac{dy}{dx}$ $\quad$ $\frac{d}{}$ $\quad$ $y = x^{1/3}$

$y^3 = x$

$(y(x))^3 = x$

$\downarrow$

$\frac{d}{dx}(y(x))^3 = \frac{d}{dx}x = 1$

$\|$

$3(y(x))^2 \cdot \frac{d}{dx}y(x)$

$3(y(x))^2 \frac{dy}{dx} = 1$

$\frac{d}{dt}(\sin(t))^3$

$= 3\sin^2(t) \cdot \cos(t)$

$$\frac{dy}{dx} = \frac{1}{3(y(x))^2}$$

$y(x) = x^{1/3}$

$\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{3(x^{1/3})^2} = \frac{1}{3}\ \frac{1}{x^{2/3}} = \frac{1}{3} x^{-2/3}$$

$$\frac{dy}{dx} = \frac{1}{3} x^{-2/3}$$

---

Find $\frac{dy}{dx}$ if $y = x^{1/3}$.

Start with $y^3 = x$

$$\frac{d}{dx} y^3 = \frac{d}{dx} x$$

$$3y^2 \cdot \frac{dy}{dx} = 1 \qquad y = x^{1/3}$$

$$y^2 = x^{2/3}$$

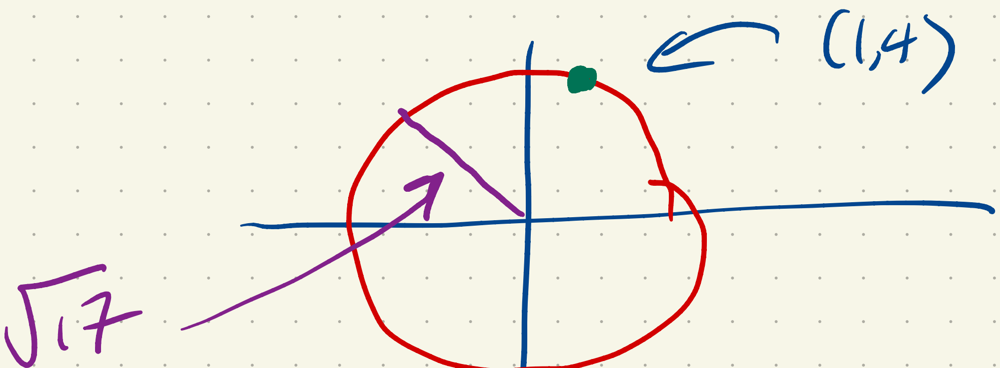
$$\frac{dy}{dx} = \frac{1}{3y^2}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{3}\frac{1}{x^{2/3}} = \frac{1}{3}x^{-2/3}}$$

---

Silly geometric fun: $y =$ $x$

Consider the curve $x^2 + y^2 = 17$




$(x,y) = (1,4)$

is on the curve.

We want to compute the equation of the tangent line to the curve at $(1,4)$

We need the slope of the tangent line.

We want $\frac{dy}{dx}$ at $x=1$ $(y=4)$

$$x^2+y^2=1$$

$$\frac{d}{dx}\left(x^2+y^2\right) = \frac{d}{dx}1$$

$$\frac{d}{dx}x^2+\frac{d}{dx}y^2 = 0$$

$\hookrightarrow (y(x))^2$

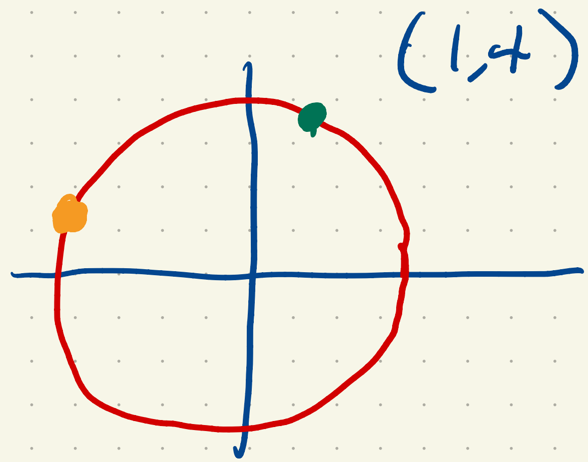
$$\frac{d}{dx}\left(y(x)\right)^2 = 2y(x)\frac{d}{dx}y(x)$$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

$$x^2 + y^2 = 17$$




$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\frac{dy}{dx} = -\frac{1}{4} \quad \text{at } (1,4)$$

$$(-4, 1) = (x, y) \qquad x^2 + y^2 = 17$$

$$\frac{dy}{dx} = -\frac{-4}{1} = +4$$

---

$$x^3 = y^4 + x^2 \sin(y) + 1$$

curve

$$\boxed{1^3 = 0^4 + 1^2 \sin(0) + 1}$$

point on the curve

$(x, y) = \underline{(1, 0)}$

What is $\frac{dy}{dx}$ at this point?

$$(x,y) = (1,0)$$

$$x^3 = y^4 + x^2\sin(y) + 1$$

$$\frac{d}{dx}x^3 = \frac{d}{dx}\left[y^4 + x^2\sin(y) + 1\right]$$

$$3x^2 = \frac{d}{dx}y^4 + \frac{d}{dx}\left(x^2\sin(y)\right) + \frac{d}{dx}1$$

$$= 4y^3\frac{dy}{dx} + \frac{d}{dx}(x^2)\cdot\sin(y) + x^2\frac{d}{dx}\sin(y)$$

$$= 4y^3\frac{dy}{dx} + 2x\sin(y) + x^2\cos(y)\frac{dy}{dx}$$

$$3x^2 = 4y^3 \frac{dy}{dx} + 2x\sin(y) + x^2\cos(y)\frac{dy}{dx}$$

$$3x^2 - 2x\sin(y) = \left[4y^3 + x^2\cos(y)\right]\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{3x^2 - 2x\sin(y)}{4y^3 + x^2\cos(y)}$$

$$x^3 = y^4 + x^2\sin(y) + 1 \qquad (1,0)$$

At $(x,y) = (1,0)$

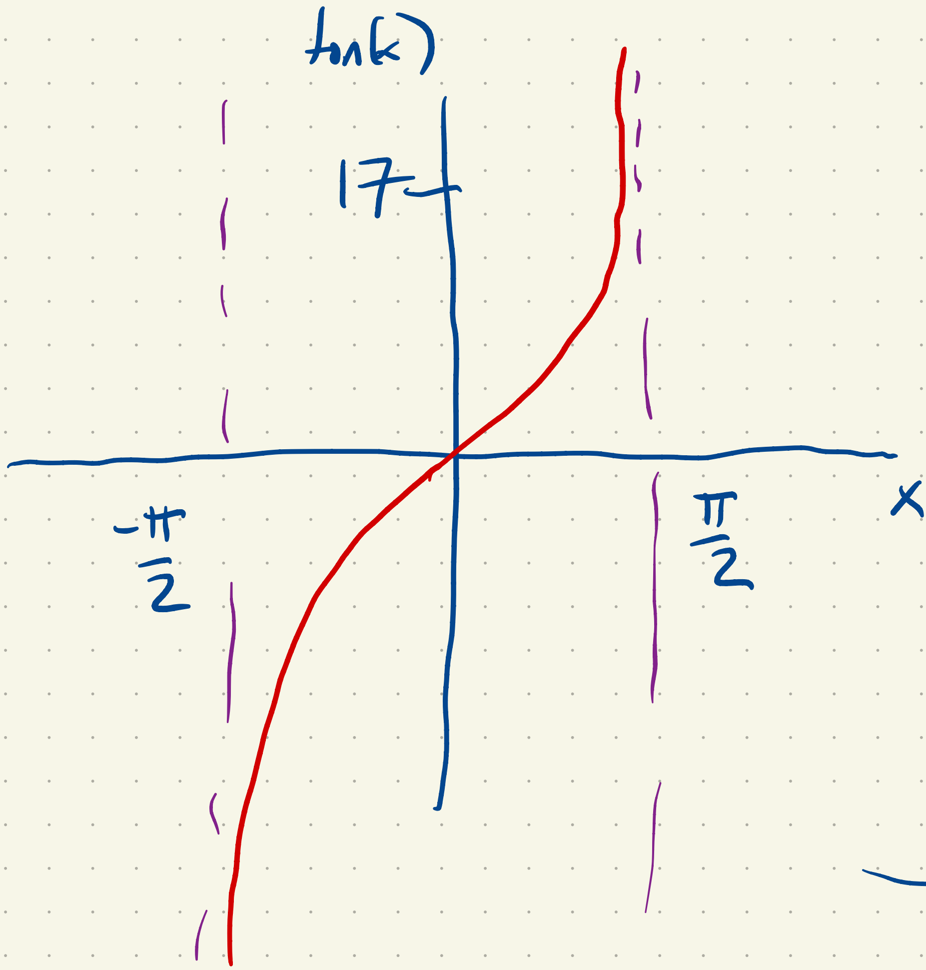
$$\frac{dy}{dx} = \frac{3\cdot 1^2 - 2\cdot 1\sin(0)}{4\cdot 0^3 + 1^2\cos(0)} = \frac{3-2\cdot 0}{0+1}$$

$$= \frac{3}{1}$$

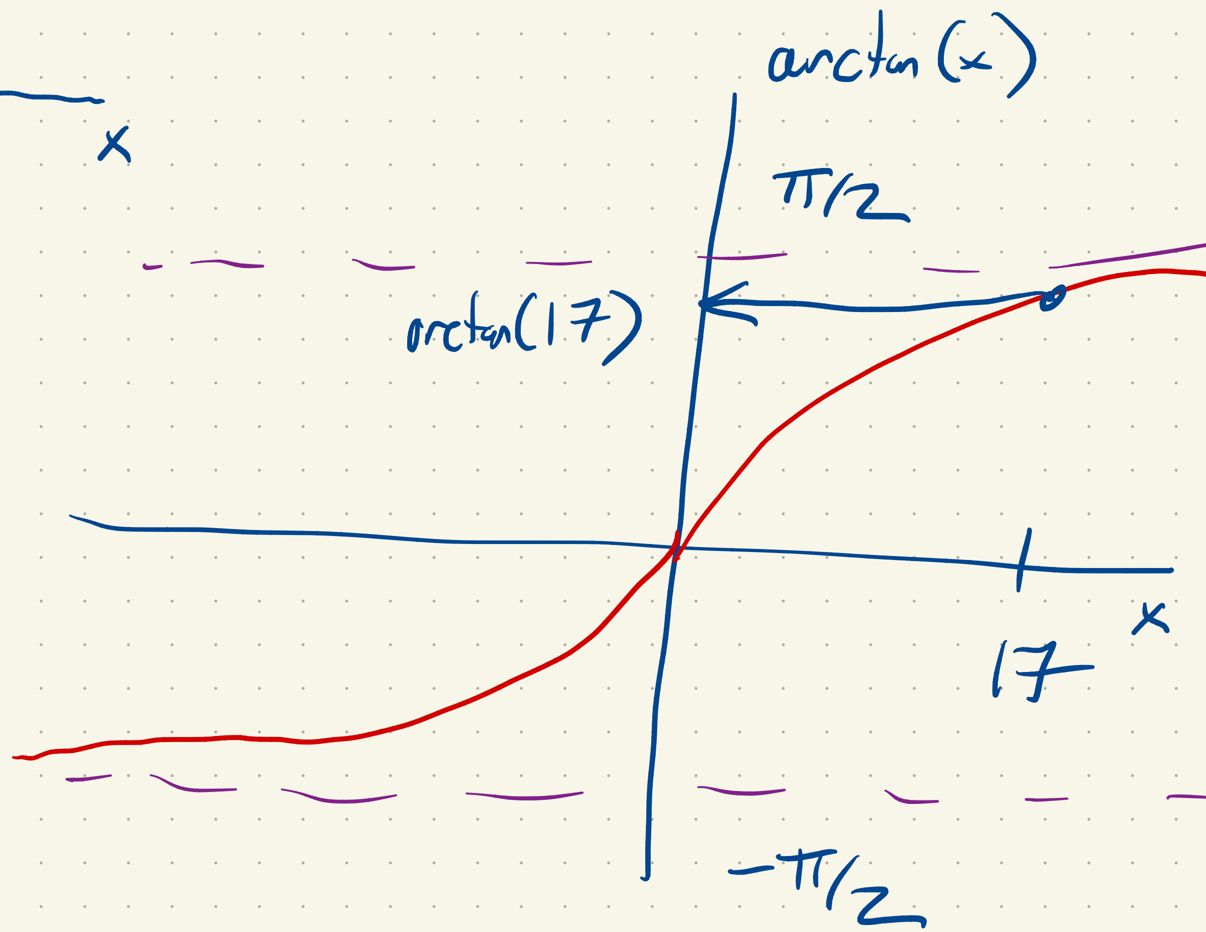
$$= 3$$

# Inverse Trig Functions

$$\frac{\sin(\theta)}{\cos(\theta)}$$




$$\tan(\theta) = 17$$



$$\tan(\arctan(z)) = z \quad \text{always}$$

$$\arctan(\tan(\theta)) = \theta \quad \text{only if}$$

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

Notation $\tan^{-1}(x) \longrightarrow \cancel{\dfrac{1}{\tan(x)}}$

$\longrightarrow$ Ugh.