Last class

$$\frac{d}{dx} x^2 = 2x$$

$$\frac{d}{dx} x^{1/2} = \frac{1}{2} x^{-1/2}$$

$$\frac{d}{dx} c = 0$$

$$\frac{d}{dx} x = 1$$

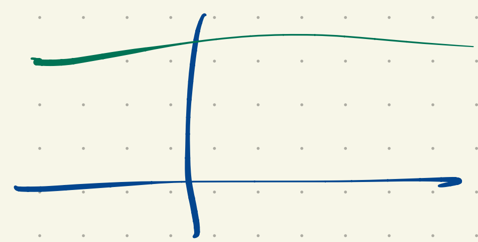
$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} 2^x = C \cdot 2^x$$

$$\frac{d}{dx} 10^x = D \cdot 10^x$$

$$\frac{d}{dx} e^7 = 0$$

$$\frac{d}{dx} 9 = 0$$



$$\frac{d}{dx}\frac{1}{x} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{x - (x+h)}{(x+h) \cdot x} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} - \frac{h}{(x+h) \cdot x} \cdot \frac{1}{h} = \lim_{h \to 0} \frac{-1}{(x+h) \cdot x}$$

$$= \frac{-1}{(x+0) \cdot x} = \frac{-1}{x^2}$$

$$\frac{d}{dx} x^{-1} = -1 \cdot x^{-2} = \frac{-1}{x^2}$$

$$\boxed{\frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2}}$$

$$\frac{d}{dx}\left(f(x)+g(x)\right) = f'(x) + g'(x)$$

$$\frac{d}{dx}\left(\underbrace{f(x)}_{x}\cdot\underbrace{g(x)}_{x}\right) = \cancel{f'(x)\cdot g'(x)}$$

$$\frac{d}{dx}\left(x\cdot x\right) \stackrel{?}{=} \left(\frac{d}{dx}x\right)\cdot\left(\frac{d}{dx}x\right)$$

$$= 1 \cdot 1 = 1$$

$$\boxed{\frac{d}{dx} x^2 = 1}$$

not true

$$\frac{d}{dx} x^2 = 2x$$

<u>Truth</u>

Product Rule

$$\frac{d}{dx} f(x)g(x) = ?$$

Product Rule

$$\frac{d}{dx} f(x)g(x)$$

$$\lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

Product Rule

$$\lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$f(x+h)g(x+h) - f(x)g(x)$$

$$= f(x+h)\left[g(x+h) - g(x)\right] + \left[f(x+h) - f(x)\right]g(x)$$

$$\boxed{-f(x+h)\,g(x) \qquad + f(x+h)\,g(x)}$$

Product Rule

$$f(x+h)\,g(x+h) - f(x)\,g(x)$$

$$= f(x+h)\left[g(x+h) - g(x)\right] + \left[f(x+h) - f(x)\right]g(x)$$

$$\lim_{h \to 0}\left(f(x+h)\left[\frac{g(x+h)-g(x)}{h}\right] + \left[\frac{f(x+h)-f(x)}{h}\right]g(x)\right)$$

$$\lim_{h \to 0} \left(f(x+h) \underbrace{\left[\frac{g(x+h) - g(x)}{h} \right]}_{g'(x)} + \underbrace{\left[\frac{f(x+h) - f(x)}{h} \right]}_{f'(x)} g(x) \right)$$

$$\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = g'(x)$$

$$\lim_{h\to 0}\left(f(x+h)\left[\frac{g(x+h)-g(x)}{h}\right]+\left[\frac{f(x+h)-f(x)}{h}\right]g(x)\right)$$

$$=\left(\lim_{h\to 0} f(x+h)\right)g'(x)+f'(x)\,g(x)$$

$$\lim_{h \to 0} \left(f(x+h) \left[\frac{g(x+h) - g(x)}{h} \right] + \left[\frac{f(x+h) - f(x)}{h} \right] g(x) \right)$$

$$= \left(\lim_{h \to 0} f(x+h) \right) g'(x) + f'(x)\, g(x)$$

$$= f(x)\, g'(x) + f'(x)\, g(x)$$

Product Rule: $\cancel{\frac{d}{dx} f(x) \cdot g(x) \neq f'(x) \cdot g'(x)}$

$$\boxed{\begin{aligned}\frac{d}{dx} f(x) g(x) &= f'(x) g(x) + f(x) g'(x) \\ &= f(x) g'(x) + f'(x) g(x)\end{aligned}}$$

$$\begin{aligned}\frac{d}{dx} x^2 = \frac{d}{dx} x \cdot x &= \left(\frac{d}{dx} x\right) \cdot x + x \cdot \left(\frac{d}{dx} x\right) \\ &= 1 \cdot x + x \cdot 1 \\ &= 2x\end{aligned}$$

$$\frac{d}{dx}x^3 = \frac{d}{dx}x \cdot x^2 = \left(\frac{d}{dx}x\right) \cdot x^2 + x \cdot \frac{d}{dx}x^2$$

$$= 1 \cdot x^2 + x \cdot 2x$$

$$= x^2 + 2x^2$$

$$= 3x^2$$

$$\frac{d}{dx}x^3 = 3x^2$$

Challenge: $\frac{d}{dx} x^4 = 4x^3$

$$\frac{d}{dx} x \cdot x^3$$

$$\frac{d}{dx} x^{n+1} = \frac{d}{dx} x \cdot x^n = (n+1) x^n$$

if $\frac{d}{dx} x^n = n x^{n-1}$

$$\frac{d}{dx}\left[(x^3-2x^2+1)e^x\right]$$

$$=\left[\frac{d}{dx}(x^3-2x^2+1)\right]e^x+(x^3-2x^2+1)\frac{d}{dx}e^x$$

$$=(3x^2-4x)e^x+(x^3-2x^2+1)e^x$$

$$=x^3e^x+x^2e^x-4xe^x+e^x$$

$$= (x^3 + x^2 - 4x + 1)e^x$$

Inverse Rule

$$\frac{d}{dx}\frac{f(x)}{g(x)}$$

$$\frac{d}{dx}\frac{1}{f(x)} = ?$$

Inverse Rule

$$\frac{d}{dx}\frac{1}{f(x)} = \lim_{h \to 0} \frac{\frac{1}{f(x+h)} - \frac{1}{f(x)}}{h}$$

$\uparrow$

Inverse Rule

$$\frac{d}{dx}\frac{1}{f(x)} = \lim_{h\to 0}\frac{\frac{1}{f(x+h)} - \frac{1}{f(x)}}{h}$$

$$= \lim_{h\to 0}\frac{f(x) - f(x+h)}{f(x)\,f(x+h)} \cdot \frac{1}{h}$$

Inverse Rule

$$\frac{d}{dx}\frac{1}{f(x)} = \lim_{h\to 0} \frac{\frac{1}{f(x+h)} - \frac{1}{f(x)}}{h}$$

$$= \lim_{h\to 0} \frac{f(x) - f(x+h)}{f(x)\,f(x+h)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-1}{f(x)f(x+h)} \cdot \left[\frac{f(x+h) - f(x)}{h}\right]$$

$$= \frac{-1}{f(x)^2} \cdot f'(x)$$

$$\boxed{\frac{d}{dx} \frac{1}{f(x)} = \frac{-f'(x)}{f(x)^2}}$$

$$\frac{d}{dx} \frac{1}{x} = \frac{-1}{x^2}$$

$$f(x) = x$$

$$\frac{d}{dx}\frac{1}{x} = \frac{-\frac{d}{dx}x}{x^2} = \frac{-1}{x^2}$$

Quotient Rule

$$\frac{f(x)}{g(x)} = f(x) \cdot \frac{1}{g(x)}$$

$$\frac{d}{dx} \frac{f(x)}{g(x)} = \left(\frac{d}{dx} f(x)\right) \frac{1}{g(x)} + f(x) \frac{d}{dx} \frac{1}{g(x)}$$

$$= \frac{f'(x)}{g(x)} + f(x) \frac{(-1)\, g'(x)}{g(x)^2}$$

$$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)\, g(x) - f(x)\, g'(x)}{(g(x))^2}$$

$$\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx} \frac{1}{f(x)} = \frac{-f'(x)}{f(x)^2}$$

$$\frac{d}{dx} \frac{e^x}{1-e^x} = \frac{e^x(1-e^x) - e^x(-e^x)}{(1-e^x)^2}$$

$$\begin{array}{l|l} f(x) = e^x & f'(x) = e^x \\ g(x) = 1-e^x & g'(x) = -e^x \end{array}$$

$$= \frac{e^x}{(1-e^x)^2}$$

$$\frac{d}{dt}\sqrt{t}\,e^t = \left(\frac{d}{dt}\sqrt{t}\right)\cdot e^t + \sqrt{t}\cdot\frac{d}{dt}e^t$$

$$= \frac{1}{2}t^{-1/2}\cdot e^t \quad + \sqrt{t}\,e^t$$

$$\frac{d}{dt}\sqrt{t} = \frac{d}{dt}t^{1/2} = \frac{1}{2}t^{-1/2}$$

$$\boxed{e^t t^{-1/2}\left(\frac{1}{2}+t\right)}$$

$$\boxed{\frac{d}{dx}f(x)g(x) = f'(x)g(x) + f(x)g'(x)}$$

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$$