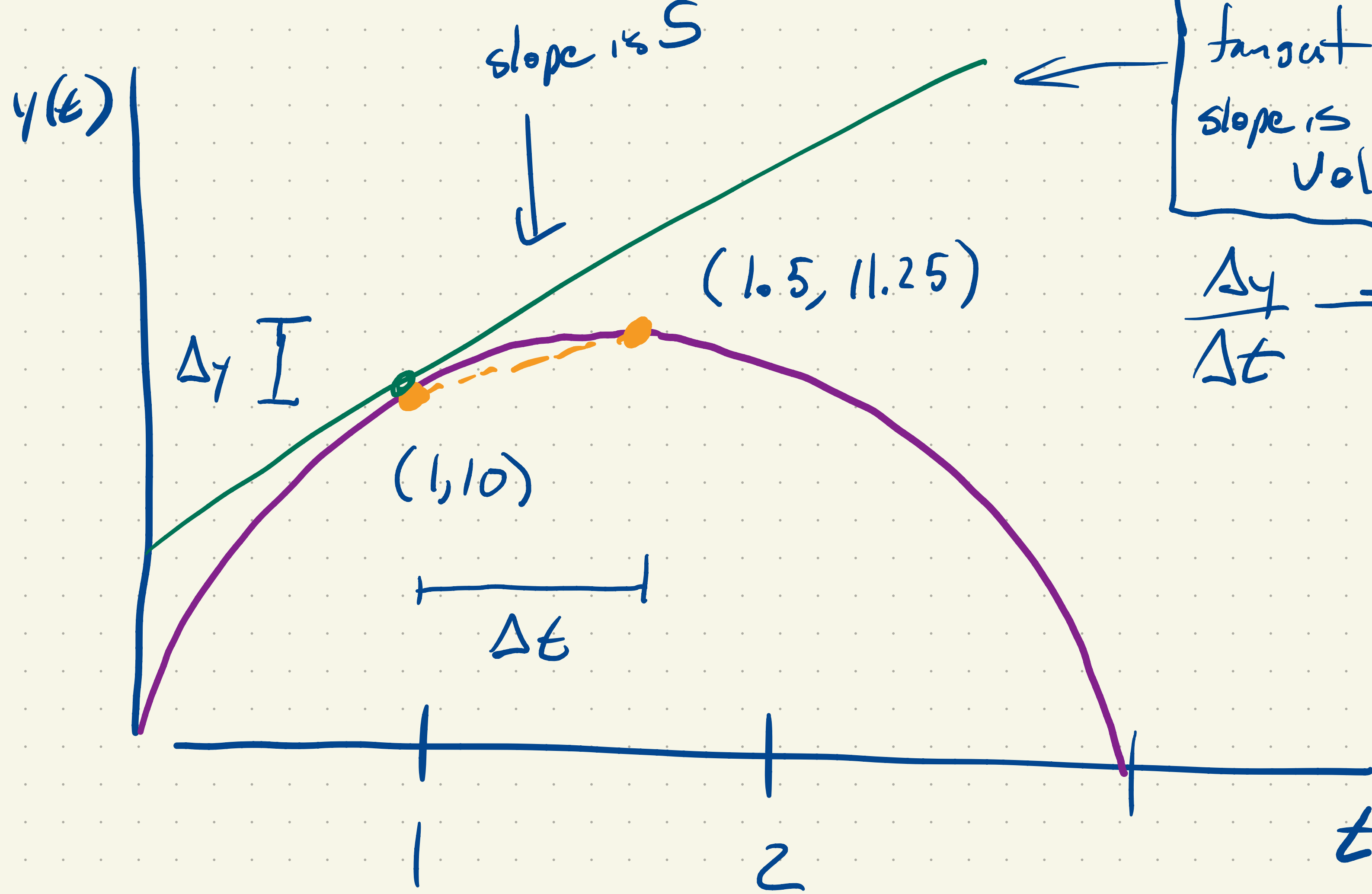
Last class

$$y(t) = 15t - 5t^2$$



tangent line
slope is instantaneous
Velocity

$$\frac{\Delta y}{\Delta t} \rightarrow \frac{m}{s}$$

Avg velocity for t in $[1, 1.5]$?

$$\Delta y = 11.25 - 10 = 1.25 \text{ m}$$

$$\Delta t = 1.5 - 1 = 0.5 \text{ s}$$

$$\frac{\Delta y}{\Delta t} = \frac{1.25 \text{ m}}{0.5 \text{ s}} = 2.5 \text{ m/s}$$

Equation of secant line?

points: $(1, 10)$, $(1.5, 11.25)$

slope: 2.5

point slope form:

$$y - y_0 = m(x - x_0)$$

$$y - y_0 = m(t - t_0)$$

$$y - 10 = 2.5(t - 1) \checkmark$$

$$y - 11.25 = 2.5(t - 1.5)$$

exercise: put these in

$y = mx + b$ form

and verify they are the

same!

$$\boxed{y = 10 + 2.5(t - 1)}$$

Derivative:

$$y'(a) = \lim_{h \to 0} \frac{y(a+h) - y(a)}{h}$$

$$\frac{\Delta y}{\Delta t} \text{ as } \Delta t \to 0$$

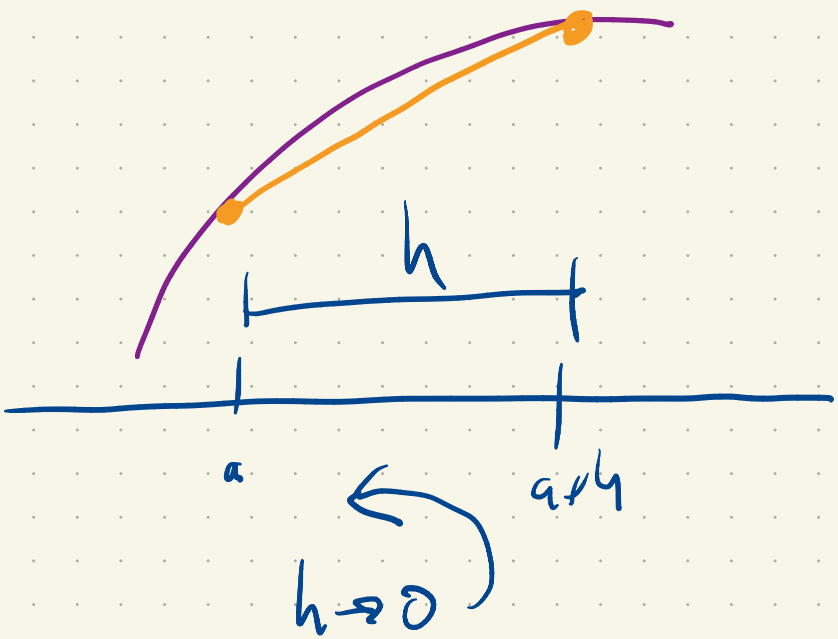
$$= \lim_{b \to a} \frac{y(b) - y(a)}{b - a}$$

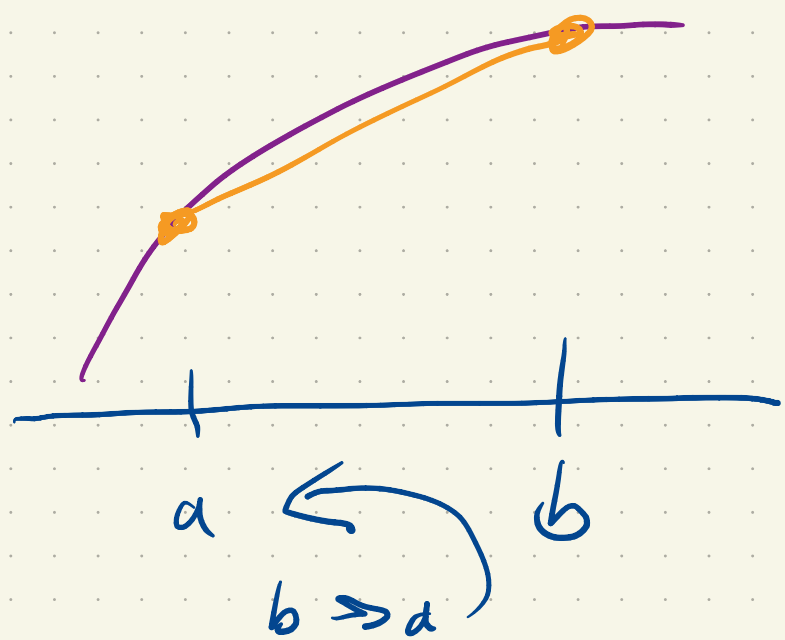
1) instantaneous rate of change

2) Slope of tangent line

$y'(1) = 5$

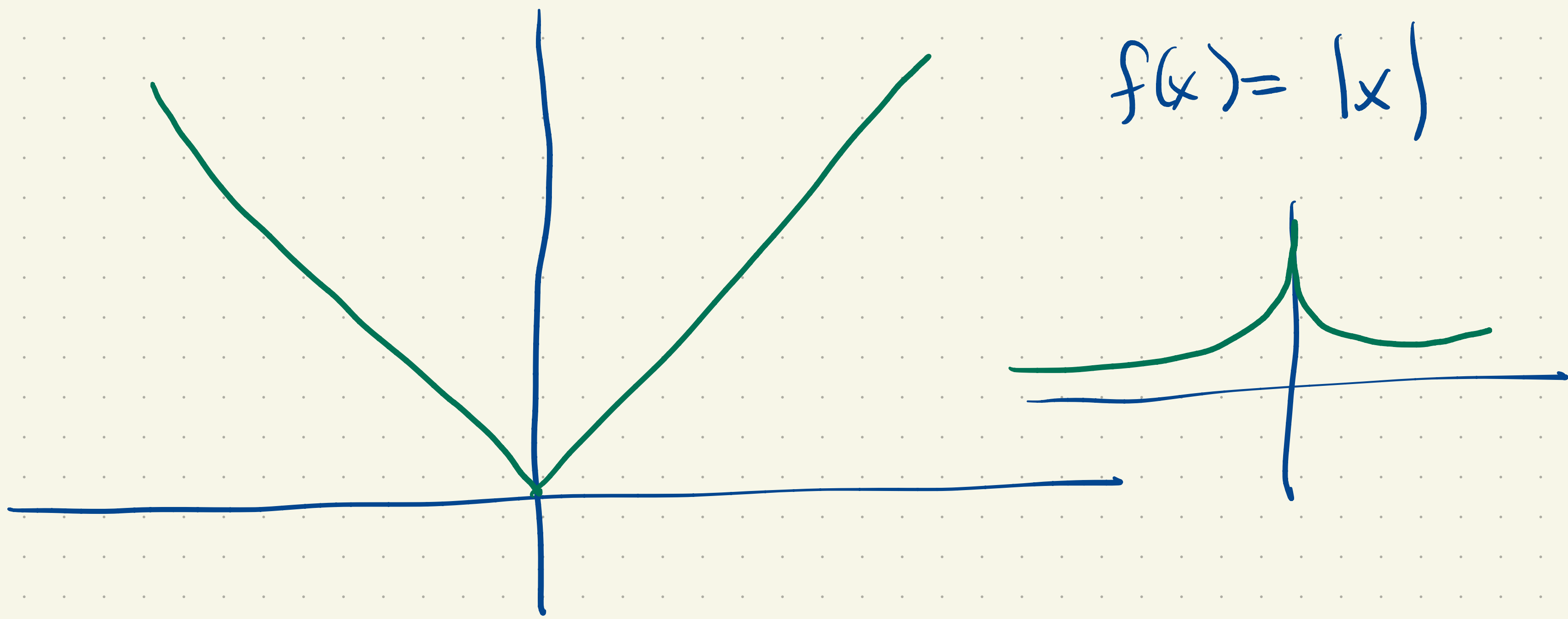
- slope of tangent line is 5 at $t=1$
- velocity is 5 m/s at $t=1$





Not every function has a derivative at every point:

$f(x) = |x|$

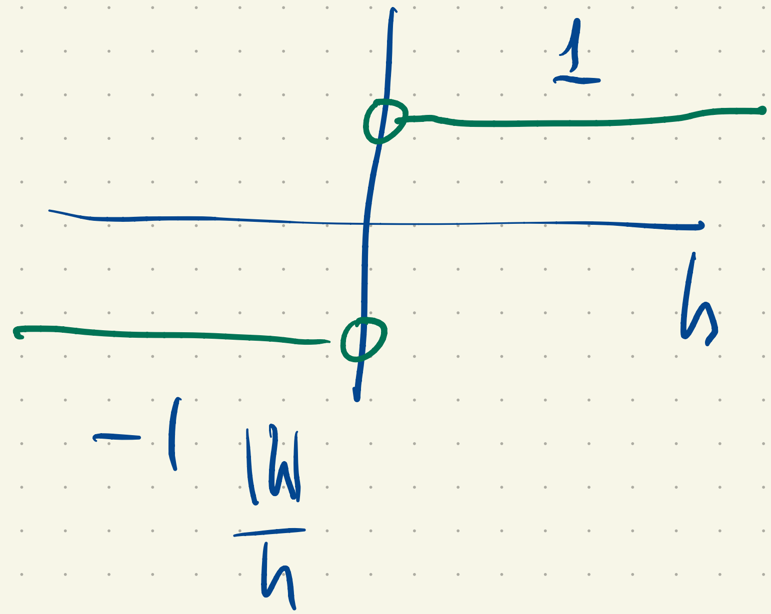


$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{|0+h| - |0|}{h}$$

$$= \lim_{h \to 0} \frac{|h|}{h}$$

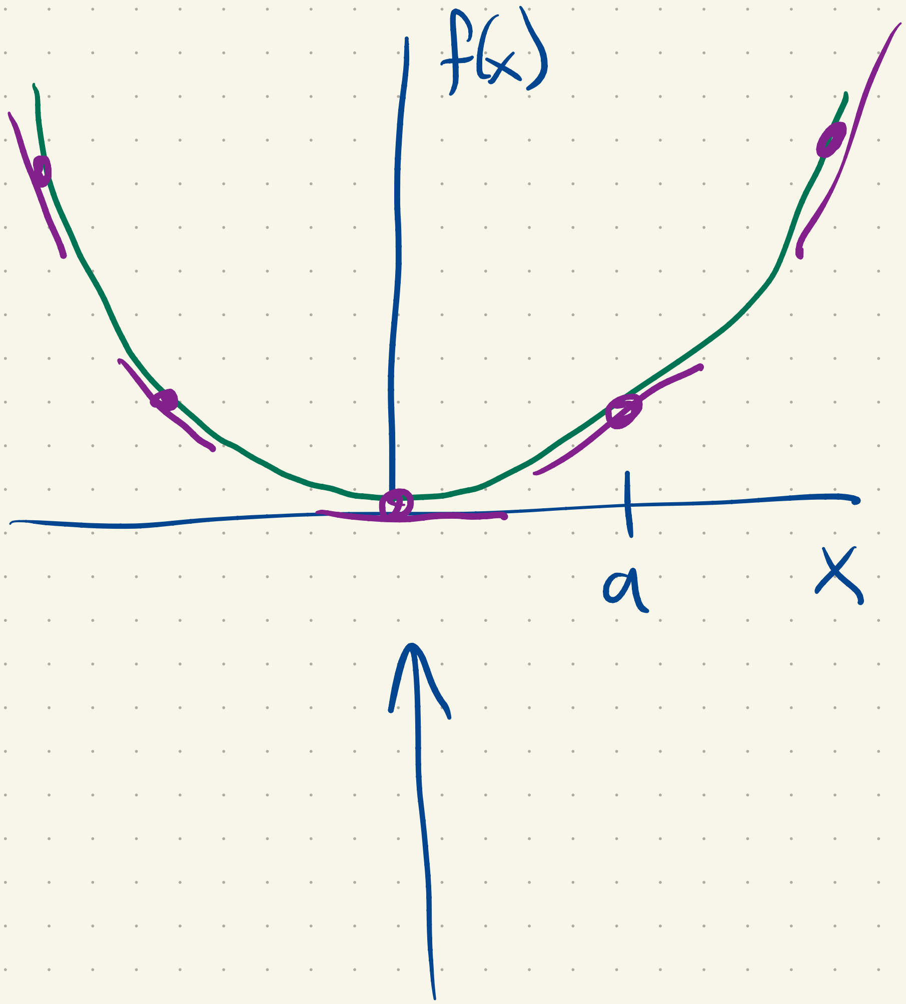
If $h > 0$, $\frac{|h|}{h} = 1$

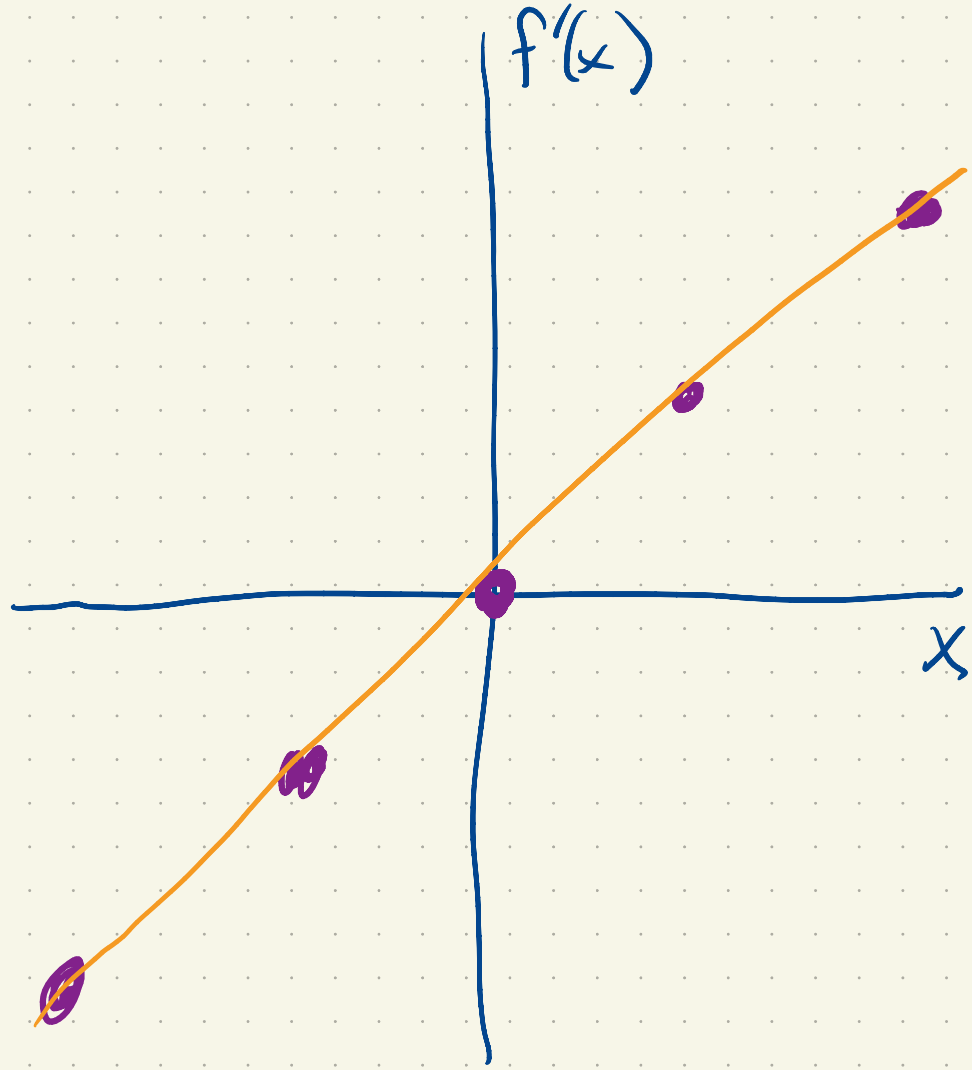
If $h < 0$ $\frac{|h|}{h} = -1$



$$\lim_{h \to 0} \frac{|h|}{h} = \text{DNE}$$

the derivative does not exist at $x=0$





$f(x) = x^2$

$f'(x) = 2x$

$\boxed{f(x) = x^3}$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$f(x) = x^2$

$a = 1$

$$= \lim_{h \to 0} \frac{(a+h)^2 - a^2}{h}$$

$$= \lim_{h \to 0} \frac{a^2 + 2ah + h^2 - a^2}{h}$$

$$= \lim_{h \to 0} \frac{2ah + h^2}{h} = \lim_{h \to 0} 2a + h$$

$$= 2a + 0$$

$$= 2a$$

$$f'(a) = 2a$$

f'(☺) = 2 ☺

$$\boxed{f'(x) = 2x}$$