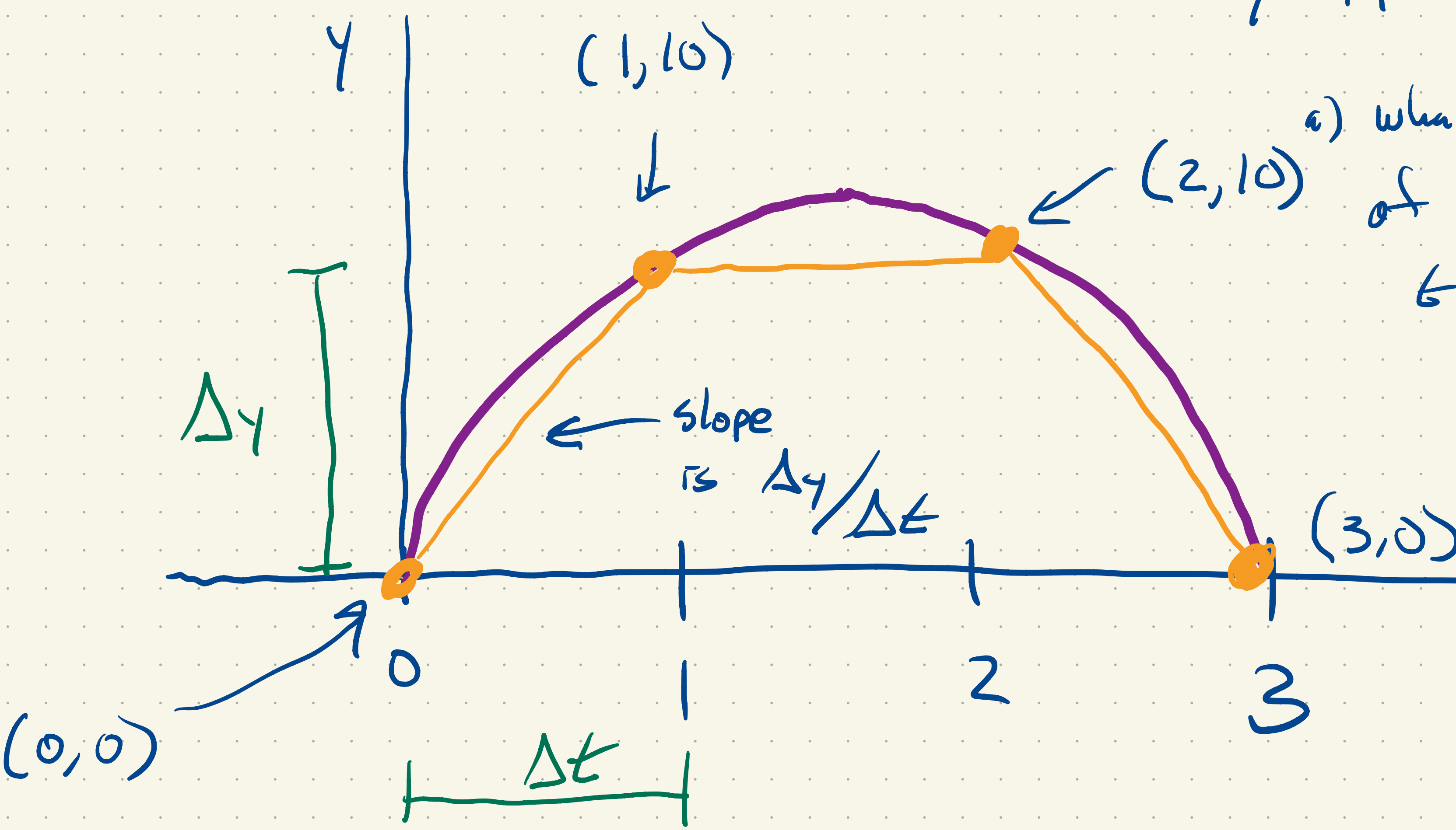
Derivatives & Rates of Change

Ball height: $y(t) = 15t - 5t^2$

t in s

y in m



a) what is height of ball at $t = 1$?

10 m

$y(1) = 10$ m

$y(0) = 0$ m

change in height from $t=0$ to $t=1$?

$\Delta y = 10$ m

Average rate of change of height from $t=0$ to $t=1$?

change in time from $t=0$ to $t=1$?

$\Delta t = 1$ s

$$\frac{\Delta y}{\Delta t} = \frac{10\text{ m}}{1\text{ s}} = 10 \text{ m/s}$$

What is the slope of line joining

(0,0) to (1,10)

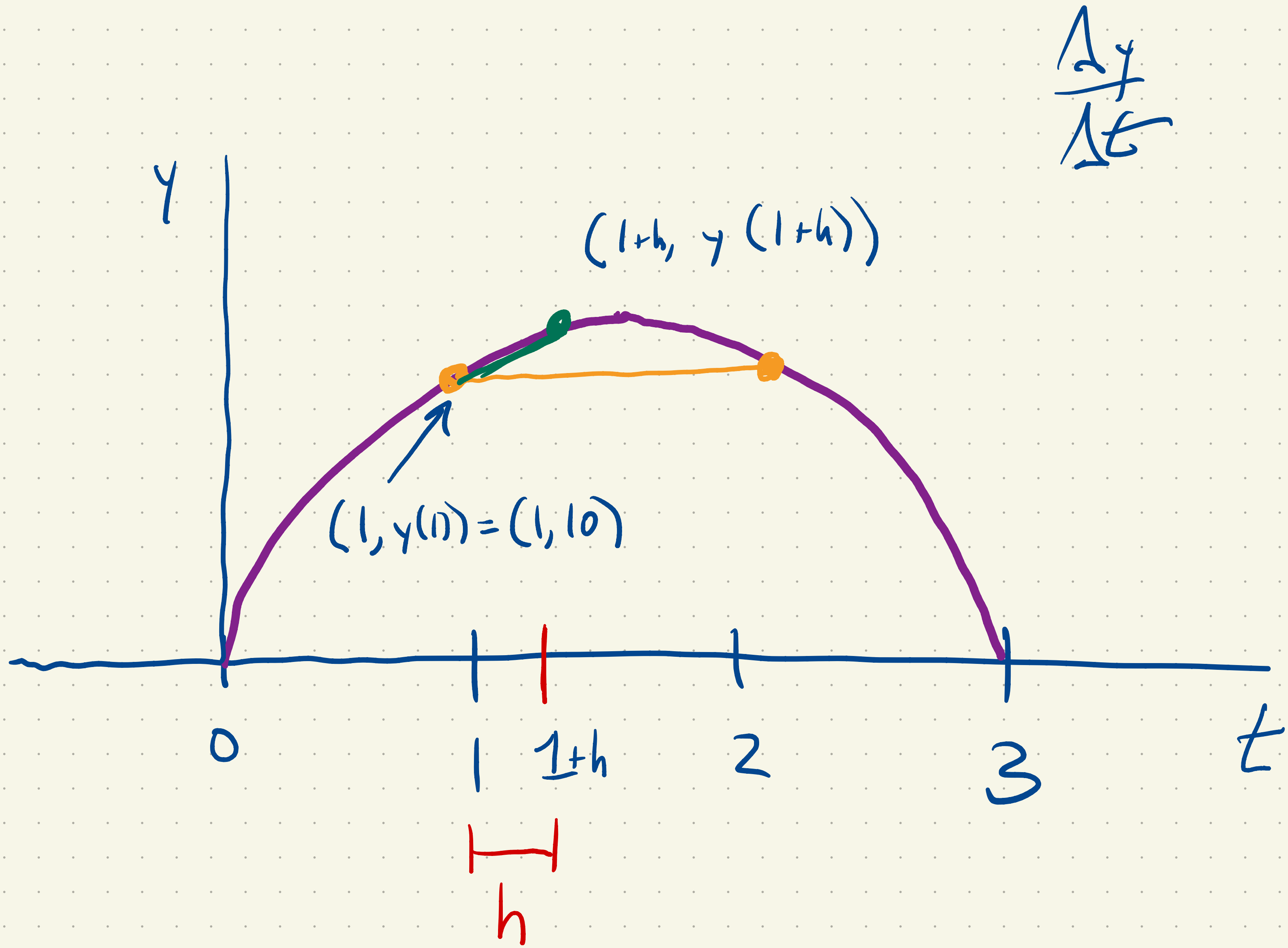
$$\frac{\Delta y}{\Delta t} = \frac{10-0}{1-0} = 10$$

Average change in height over [1,2]

$$\frac{\Delta y}{\Delta t} \quad \frac{y(2) - y(1)}{2 - 1} = \frac{10 - 10}{1} = \frac{0}{1} = 0 \ \frac{m}{s}$$

Avenge chnge in height over $[2,3]$

$$\frac{\Delta y}{\Delta t} = \frac{y(3) - y(2)}{3 - 2} = \frac{0 - 10}{3 - 2} = \frac{-10}{1} = -10\frac{m}{s}$$



Avg rate of change of height over $[1, 1+h]$

$$\Delta y = y(1+h) - y(1)$$

$$y(t) = 15t - 5t^2$$

$$\Delta t = 1+h - 1 = h$$

$$y(1) = 10$$

$$y(1+h) = 15(1+h) - 5(1+h)^2$$

$$= 15 + 15h - 5(1+2h+h^2)$$

$$= 10 + 5h - 5h^2$$

$$\Delta y = y(1+h) - y(1) = 5h - 5h^2$$

$$\Delta t = h$$

h: length of time interval

$$\frac{\Delta y}{\Delta t} = \frac{5h - 5h^2}{h}$$

$h = 1$ $\quad \frac{\Delta y}{\Delta t} = 0$

$$\lim_{h \to 0} \frac{5h - 5h^2}{h} = \lim_{h \to 0} 5 - 5h = 5 - 5 \cdot 0$$

$$= 5 \ \frac{m}{s}$$

Given a time a in the interval

$$\frac{\Delta y}{\Delta t} \qquad \frac{y(a+h)-y(a)}{h}$$

← avg rate of change of y over $[a, a+h]$.

h ↓ $(a+h-a)=h$

$[a, a+h]$

↑ $a=1$ in above

$$y'(a)=\lim_{h\to 0}\frac{y(a+h)-y(a)}{h}$$

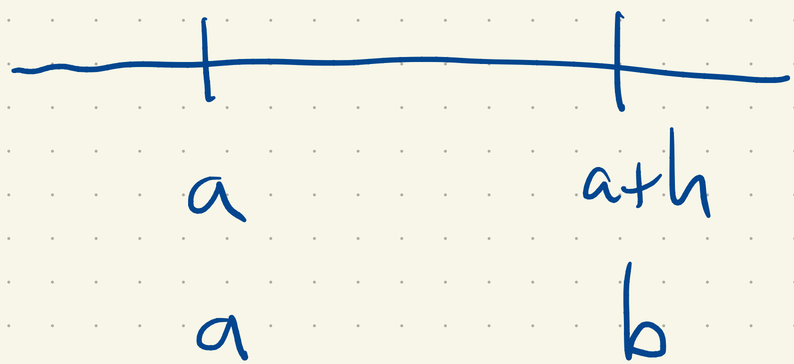
↳ the derivative of y w.r.t t at $t=a$.

Two interpretations

1) instantaneous rate of change of y w.r.t. t at $t = a$

2) slope of tangent line to graph at $t = a$.

Alternative form:



$$\frac{\Delta y}{\Delta t} \qquad \frac{y(b) - y(a)}{b - a}$$

← avg. rate of change over $[a, b]$

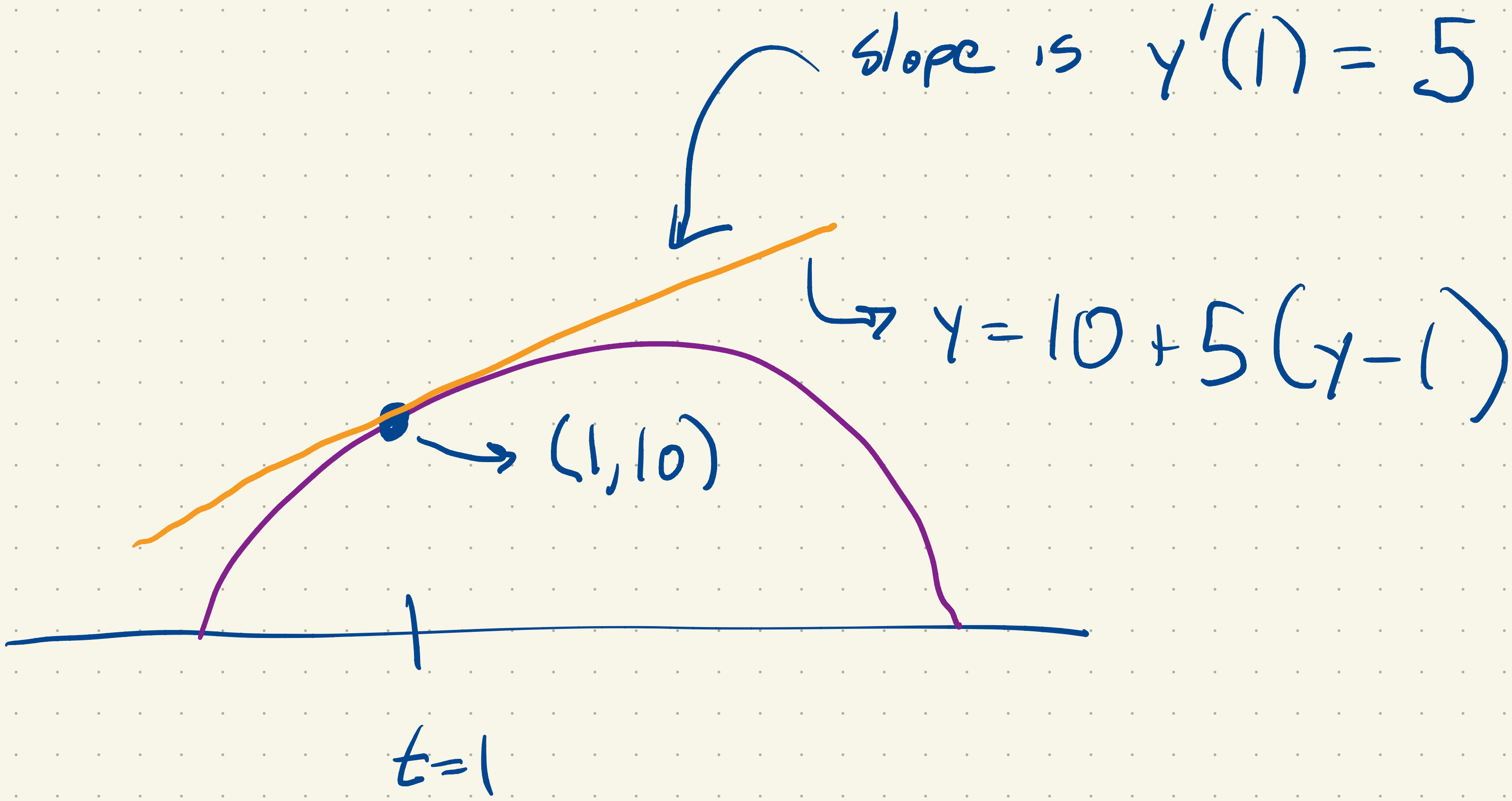
$[a, a+h]$

$$\lim_{b \to a} \frac{y(b) - y(a)}{b - a}$$

$$\Updownarrow$$

$$\lim_{h \to 0} \frac{y(a+h) - y(a)}{h}$$

What is the equation of tangent line at $t=1$?



$y = mx + b$

Point slope (x_0, y_0) point

m slope

$$y - y_0 = m(x - x_0)$$

$$y = y_0 + m(x - x_0)$$

$$(t_0, y_0) = (1, 10)$$

$$y = y_0 + m(t - t_0)$$

$$m = 5$$

$$y = 10 + 5(t - 1)$$ ← eq of tangent line.