Why limits? $\frac{0}{0}$, $\frac{0}{1} = 0$, $\frac{1}{0}$

$$\frac{\Delta x}{\Delta t} \to \frac{0}{0}$$

Continuity (2.5)

$\hookrightarrow$ Direct Subs. Property.

$$\lim_{x \to 3} x^2 - 2x + 1 = 3^2 - 2 \cdot 3 + 1 = 4$$

A function $f(x)$ is continuous at some a

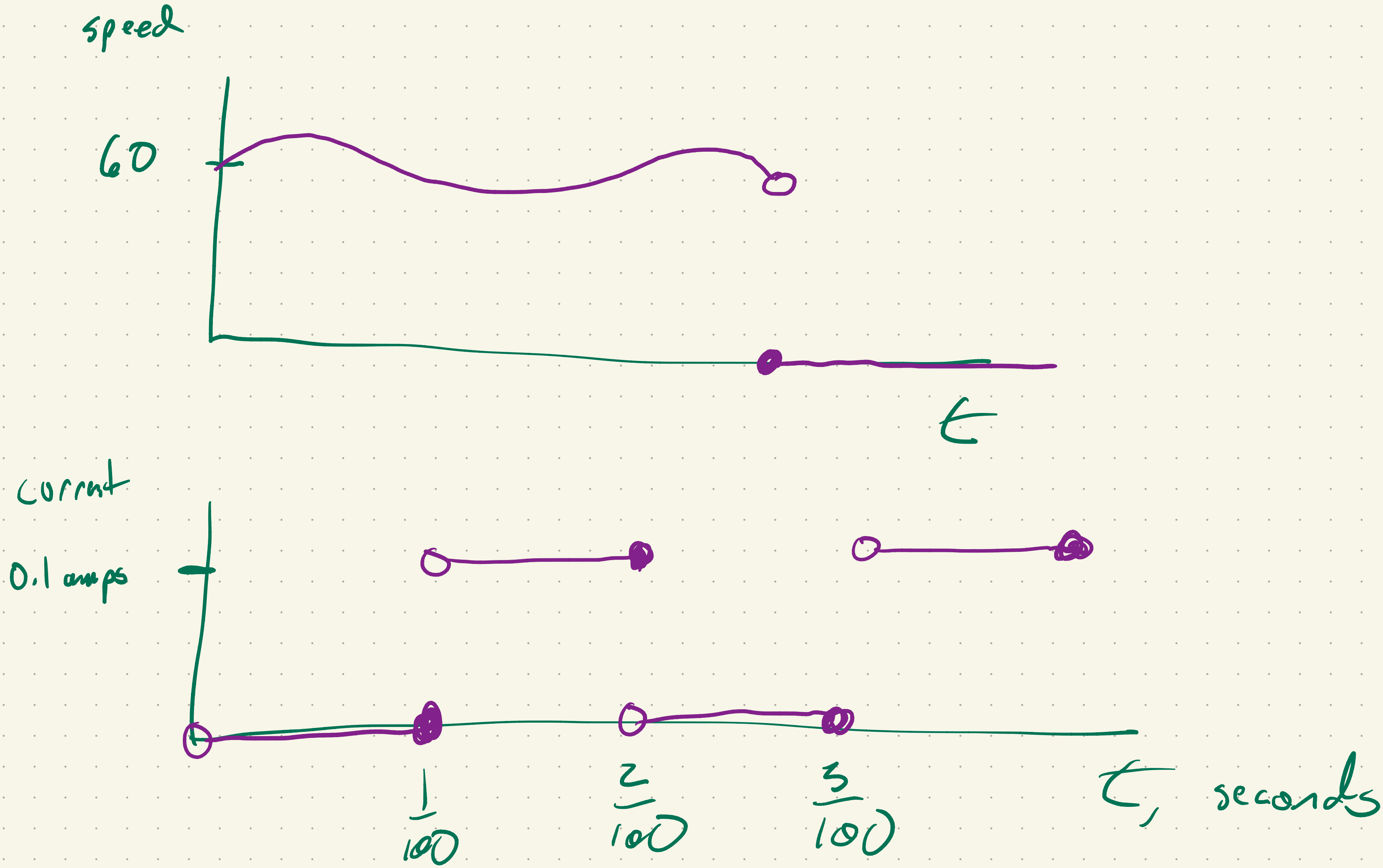
in its domain if

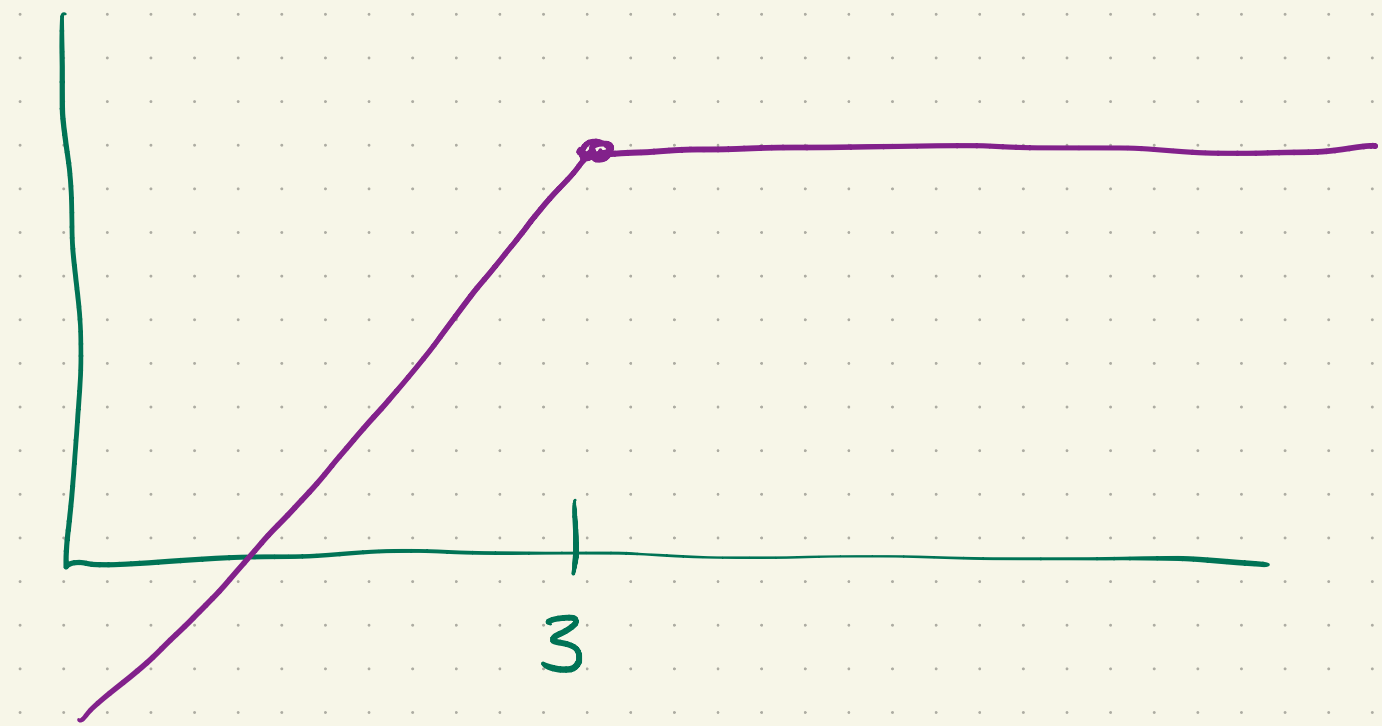
$$\lim_{x \to a} f(x) = f(a)$$

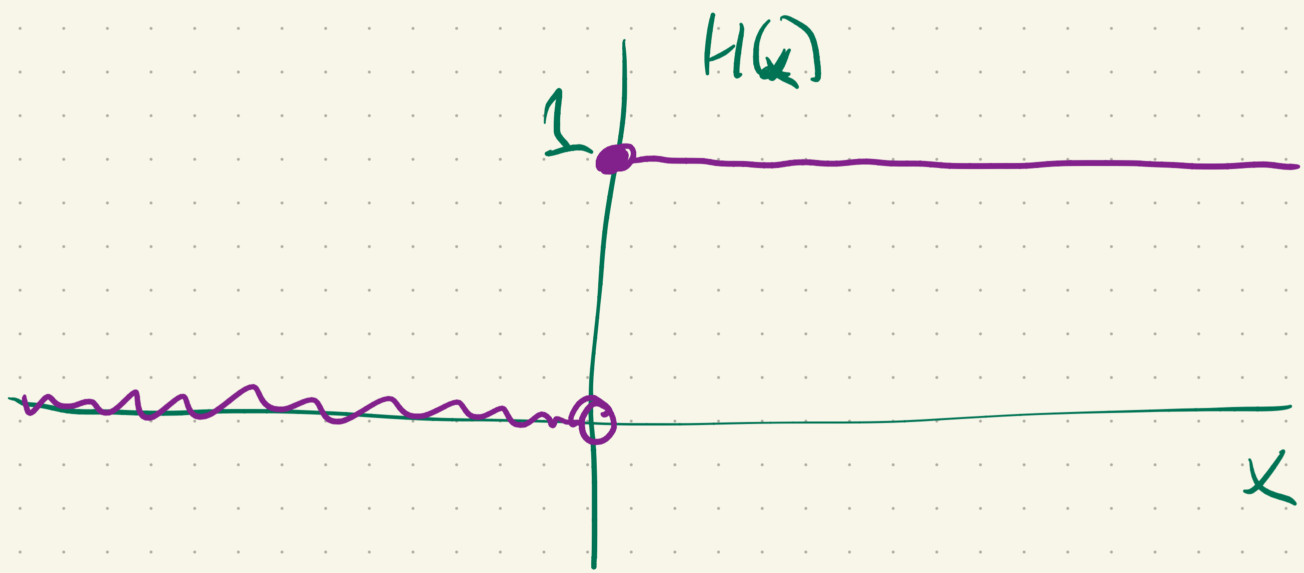
A function is continuous if it is continuous at each point in its domain

Not continuous: discontinuous

What does discontinuity look like?


speed
60
t
current
0.1 amps
1/100
2/100
3/100
t, seconds


3


H(x)
1
x

$$\lim_{x \to 0} H(x) = H(0) \; ?$$

$$\rightarrow H(0) = 1$$

But $\lim_{x \to 0} H(x)$ does not exist.

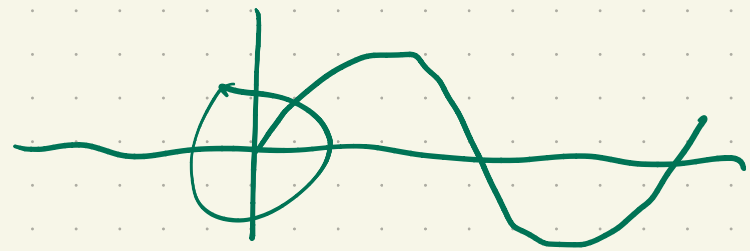
$$\lim_{x \to 0^-} H(x) = \lim_{x \to 0^-} 0 = 0$$

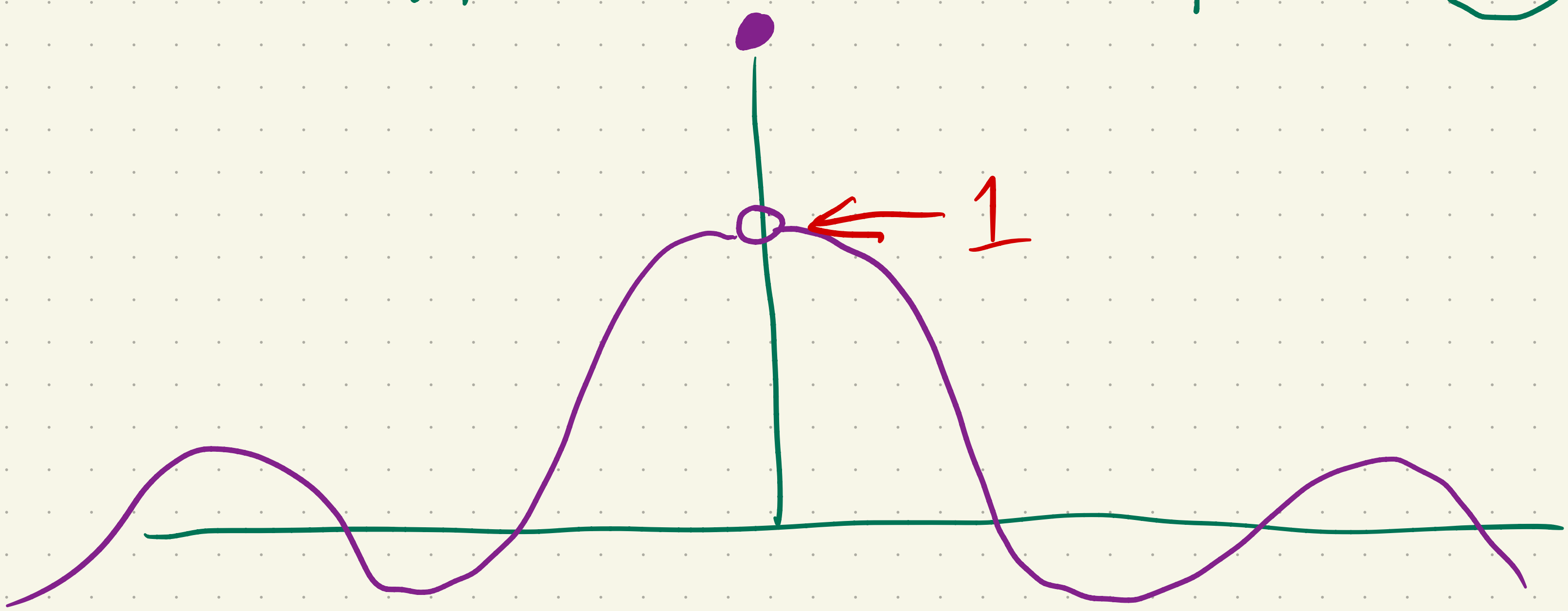
$$\lim_{x \to 0^+} H(x) = \lim_{x \to 0^+} 1 = 1$$

If left and right limits disagree, then limit does not exist.

$\sin(0)$

$$f(x) = \frac{\sin(x)}{x}$$





$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

$$g(x) = \begin{cases} \frac{\sin(x)}{x} & x \neq 0 \\ 2 & x = 0 \end{cases}$$

Is $g(x)$ continuous at $x = 0$?

$$\lim_{x \to 0} g(x) = g(0) \ ?$$

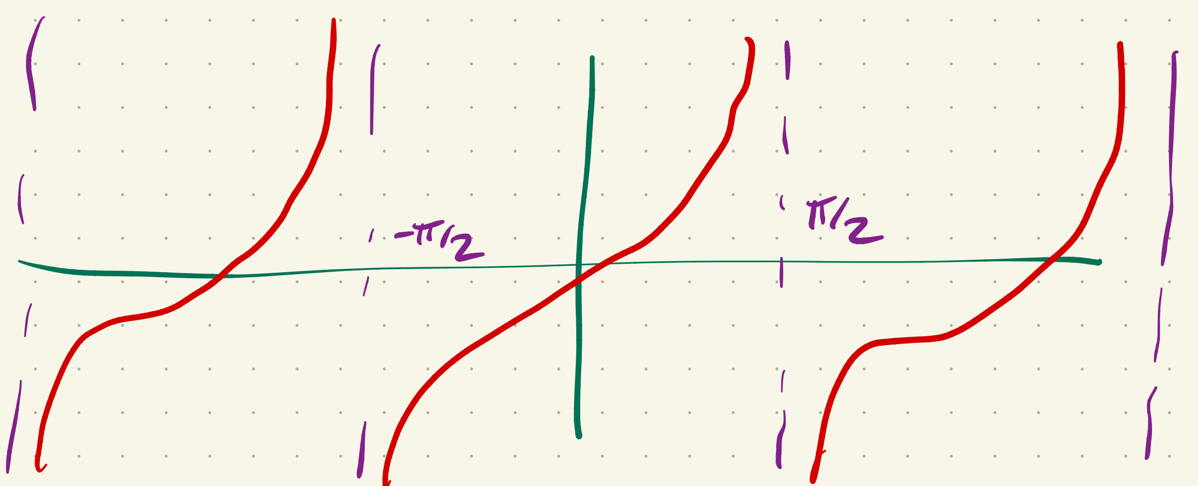
$$g(0) \to 2$$

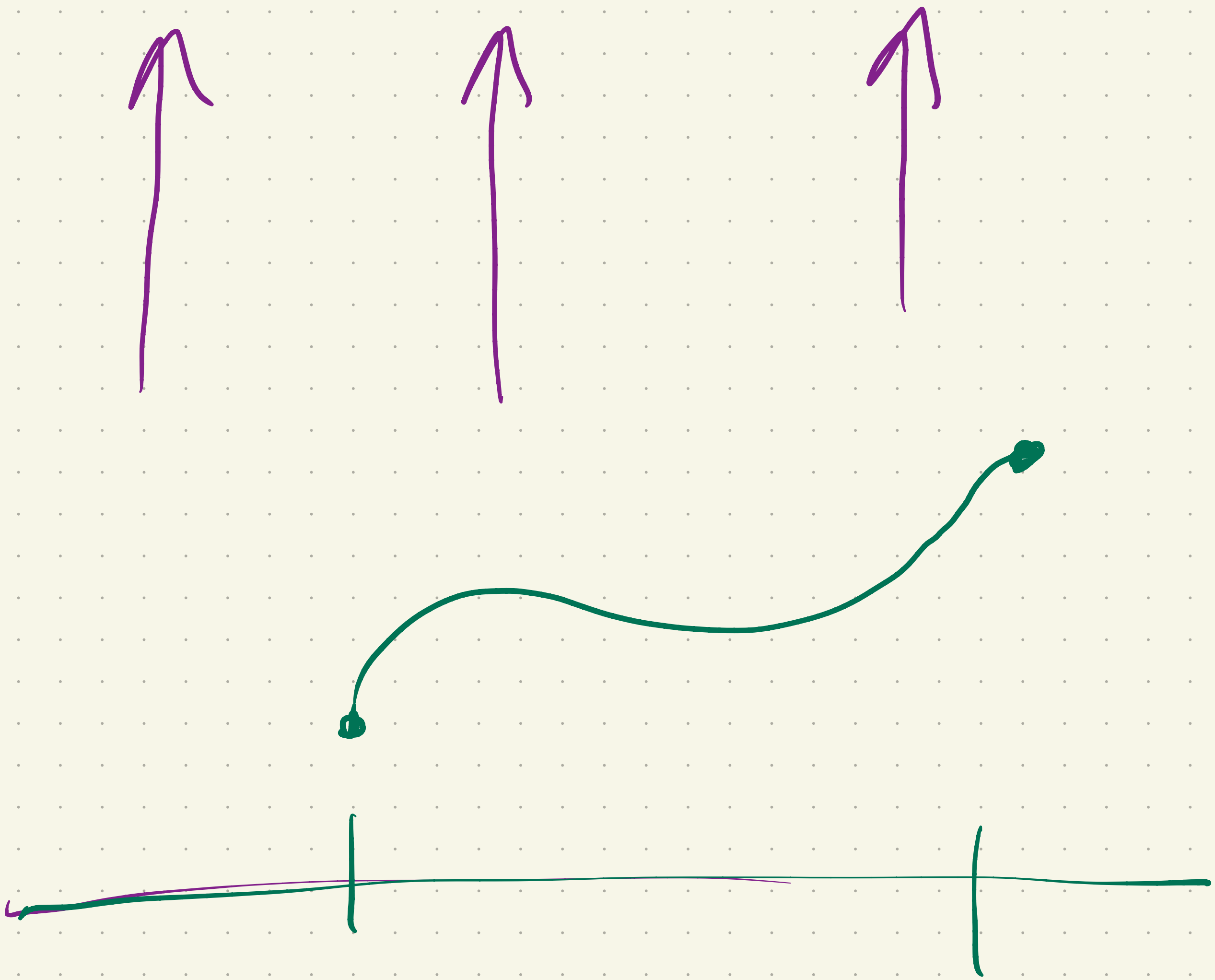
$$\lim_{x \to 0} g(x) = \lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

$1 \neq 2$, so not continuous

The trig functions $\sin(x)$, $\cos(x)$ are continuous.

What about $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$





Following functions are cts: poly'ls (on their
rational
roots domains)
trig
exp
log

$\sin(x^2+3)$ ~~a~~ composition of continuous functions is continuous.

$+, -, *$

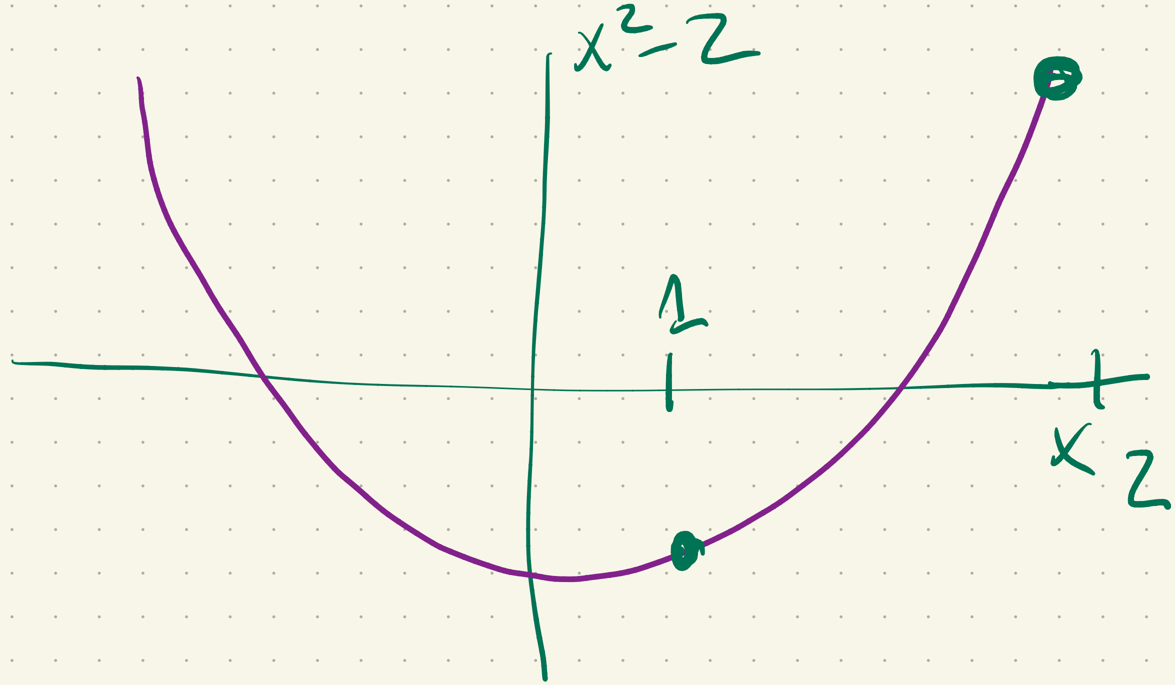
Is there a number x with $x^2 = 2$?

$\sqrt{2} \cdot \sqrt{2} = 2$

$f(x) = x^2 - 2$

$f(\sqrt{2}) = (\sqrt{2})^2 - 2 = 2 - 2 = 0$

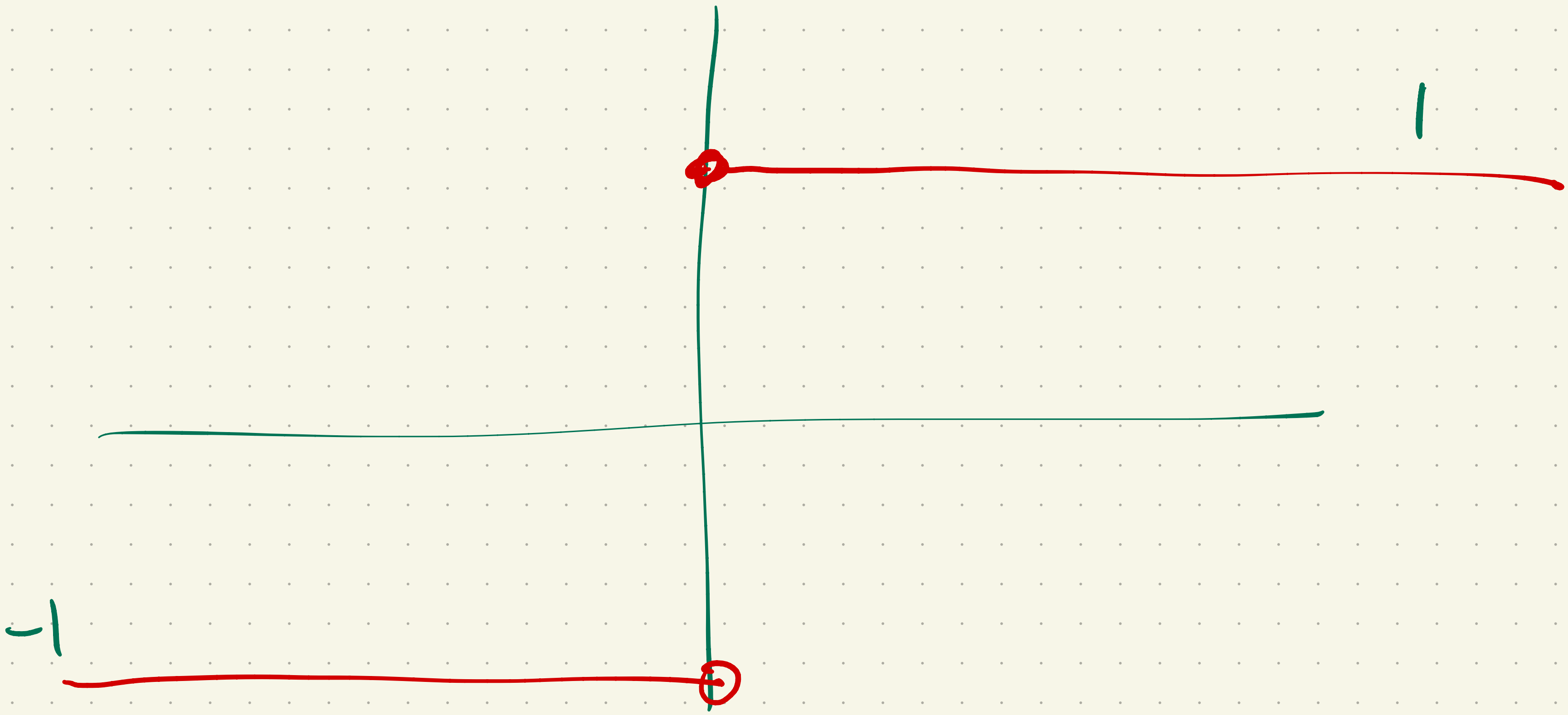
$f(x) = x^2 - 2$



$f(0) = -2$ $\qquad$ $f(2) = 2$

$f(1) = -1$

If $f(1) < 0$ and $f(2) > 0$ there should be a spot x between 1 and 2 where $f(x) = 0$


1
-1

Intermediate Value Theorem

If $f(x)$ is continuous on $[a,b]$

and if y is a number between $f(a)$ and $f(b)$

then there exists x in $[a,b]$ where

$$f(x) = y.$$