Limits: $\frac{0}{0}$

↳ leads to "holes" in graphs

→ average rate of change → instantaneous rate of change

$$\frac{\Delta x}{\Delta t}$$

$\Delta x = 0$

$\Delta t = 0$

$\frac{0}{0}$

$\frac{1}{0}$ → limits help here too

$$\lim_{x \to 3^+} \frac{1-x}{3-x} = \frac{-2}{0^-} = +\infty$$

$3 - 3.1$

$$\frac{-2}{-0.01} = 200$$

$$f(x) = \frac{1-x}{3-x}$$

$$\lim_{x \to 3^+} f(x) = +\infty$$

Rules for working with limits:

$$\lim_{x \to a} f(x) = L \qquad \lim_{x \to a} g(x) = M$$

Then

$$\lim_{x \to a} \left(f(x) + g(x) \right) = L + M$$

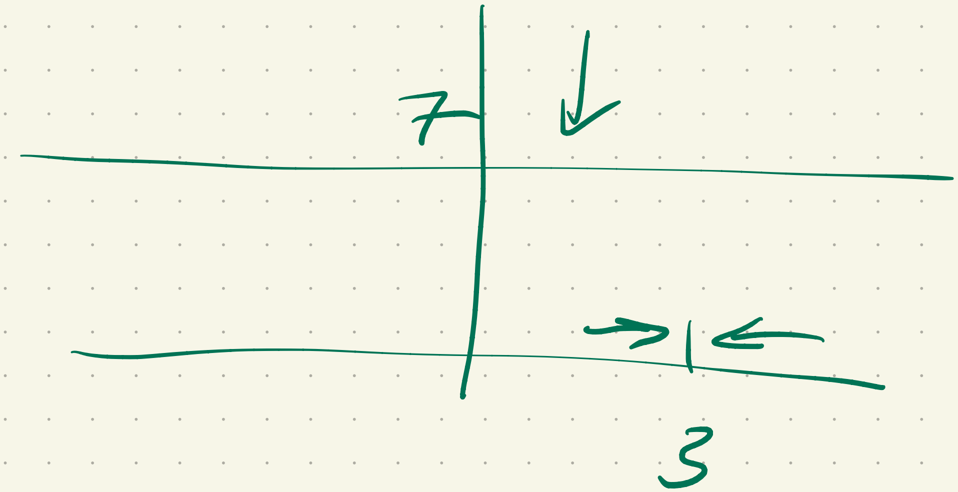
$$\lim_{x \to a} \left(f(x) - g(x) \right) = L - M$$

$$\lim_{x \to a} f(x)\, g(x) = L \cdot M$$

$$\lim_{x \to a} f(x)g(x) = \left(\lim_{x \to a} f(x)\right) \cdot \left(\lim_{x \to a} g(x)\right)$$

Division is interesting!

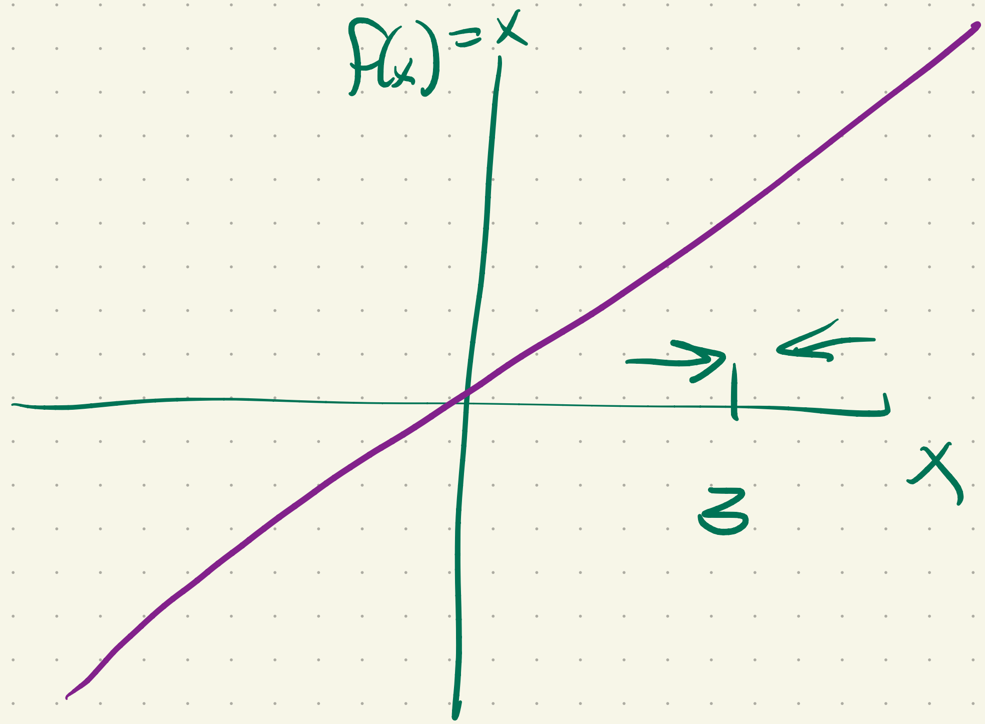
$$\lim_{x\to 3} 7 = 7$$



$$\lim_{x\to a} c = c$$

$$\lim_{x\to 3} x = 3$$

$f(x) = x$



$$\boxed{\begin{aligned}&\lim_{x \to a} x = a\\ &\lim_{x \to a} c = c\end{aligned}}$$

$+(-2x)$

$$x^2 = x \cdot x$$

$$\lim_{x \to a} x^2 - 2x + 3 = \lim_{x \to a} x^2 - \lim_{x \to a} 2x + \lim_{x \to a} 3$$

$$= \left(\lim_{x \to a} x\right)\left(\lim_{x \to a} x\right) - \left(\lim_{x \to a} 2\right) \cdot \left(\lim_{x \to a} x\right)$$

$$+ \lim_{x \to a} 3$$

$$= a \cdot a - 2 \cdot a + 3$$

$$= a^2 - 2a + 3$$

$$\lim_{x \to a} x^2 - 2x + 3 = a^2 - 2a + 3$$

DSP

Direct Substitution Property

$$\lim_{x \to a} f(x) = f(a)$$

Every polynomial has the direct subs property.

Lots of functions have DSP

$$\lim_{x \to a} x^{1/n} = a^{1/n}$$

Division:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)}$$

so long as $\lim\limits_{x \to a} g(x) \neq 0$.

$x^3 - 7x + 1$

$x^5 + 2x + 1$

$$\lim_{x \to 2} \frac{1-2x}{3x^2+1} = \frac{\lim\limits_{x \to 2} 1-2x}{\lim\limits_{x \to 2} 3x^2+1}$$

$$= \frac{1-2\cdot 2}{3\cdot 2^2+1}$$

$$= \frac{-3}{13} \leftarrow \neq 0$$

justification

Rational functions have

DSP. on

their domain.

"Limits don't care about one point"

If $f(x) = g(x)$ except at $x = a$

and if $\lim_{x \to a} g(x) = L$ then

$$\lim_{x \to a} f(x) = L \quad \text{also.}$$

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} \frac{(x-1)(x+1)}{x-1}$$

limits don't care

$$\frac{0}{0}$$

$$= \lim_{x \to 1} x + 1$$

direct substitution

$$= 1 + 1$$

$$= 2$$

$$\frac{x^2-1}{x-1}$$

$\Updownarrow$

$x+1$