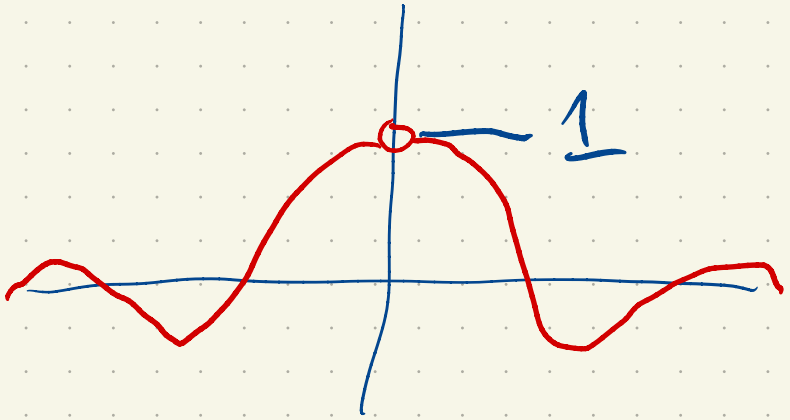
Last class:

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

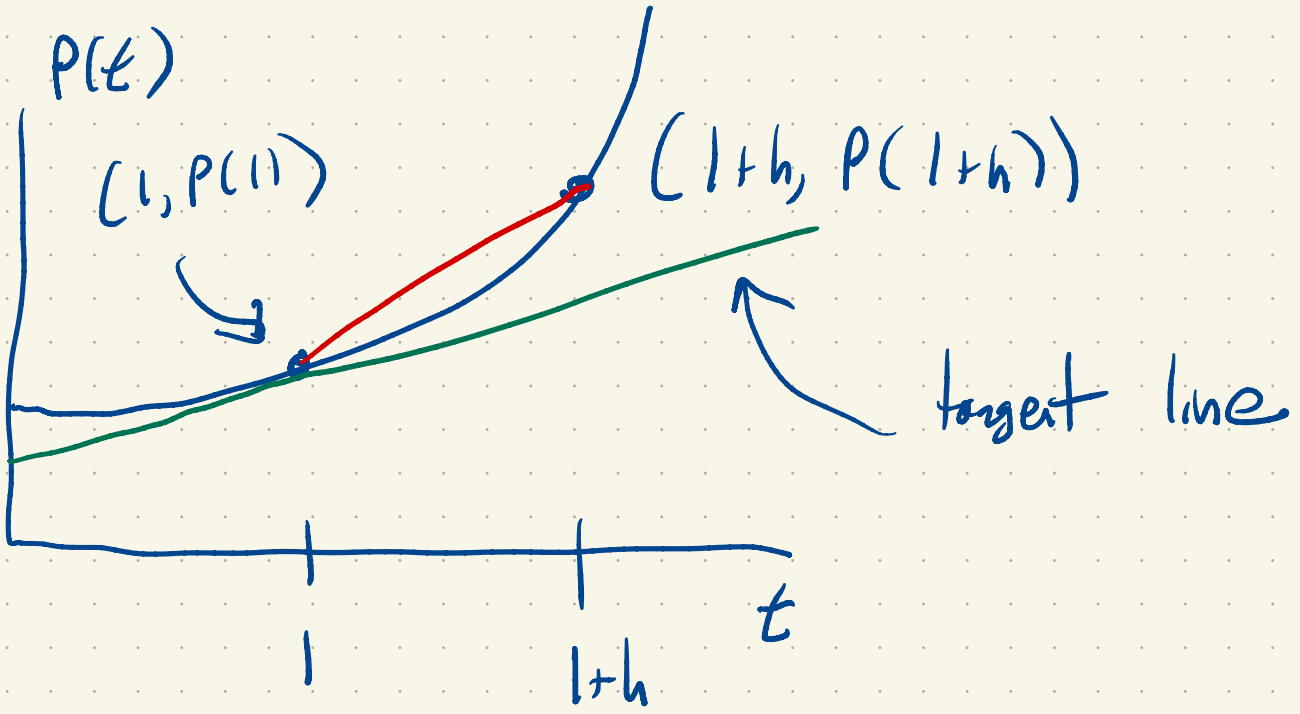
$\frac{0}{0}$



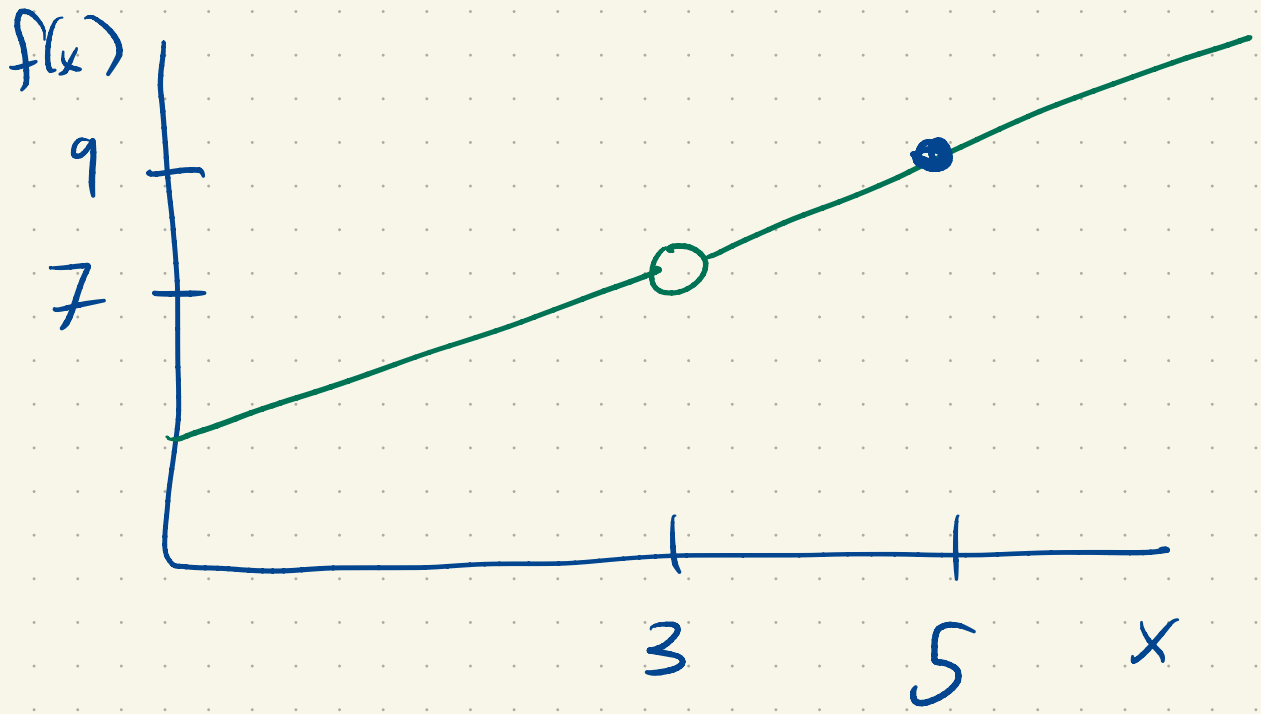
$$\lim_{h \to 0} \frac{d(t_0+h) - d(t_0)}{h}$$ → instantaneous speed from average speeds over short time intervals

$[t_0, t_0+h]$

$$\lim_{h \to 0} \frac{P(1+h) - P(1)}{h}$$ instantaneous population growth rate



$$\lim_{h \to 0} \frac{P(1+h) - P(1)}{h}$$: slope of tangent line



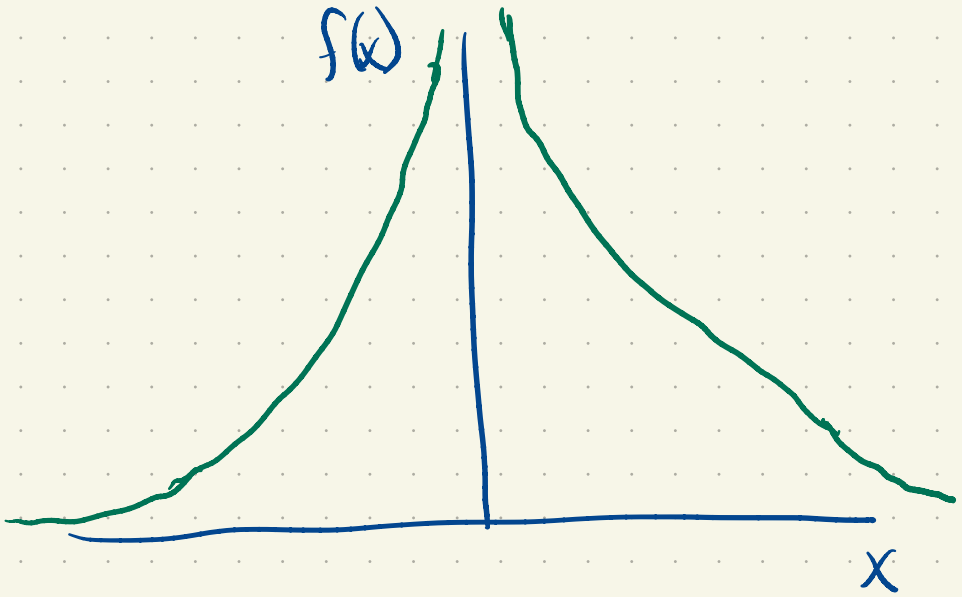
$$\lim_{x \to 3} f(x) = 7 \qquad \lim_{x \to 5} f(x) = 9$$

Variations:

$$\frac{0}{0} \longrightarrow \frac{7}{0} \stackrel{?}{=} \pm\infty \qquad \frac{1}{0}$$

$$\frac{7}{0.01} = 700$$

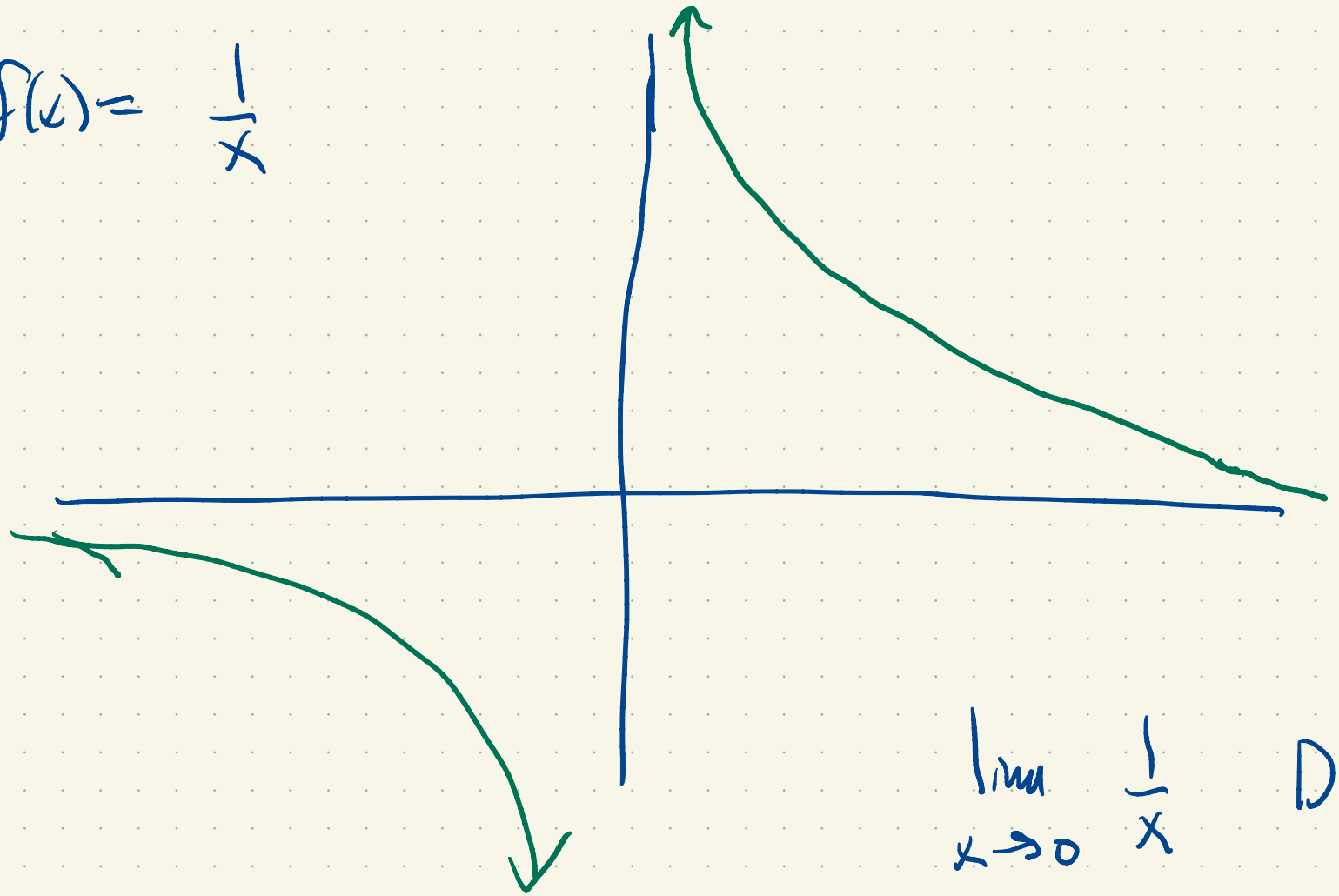
$$\frac{7}{0.001} = 7000$$



$$f(x) = \frac{1}{x^2}$$

$$\lim_{x \to 0} \frac{1}{x^2} = +\infty$$

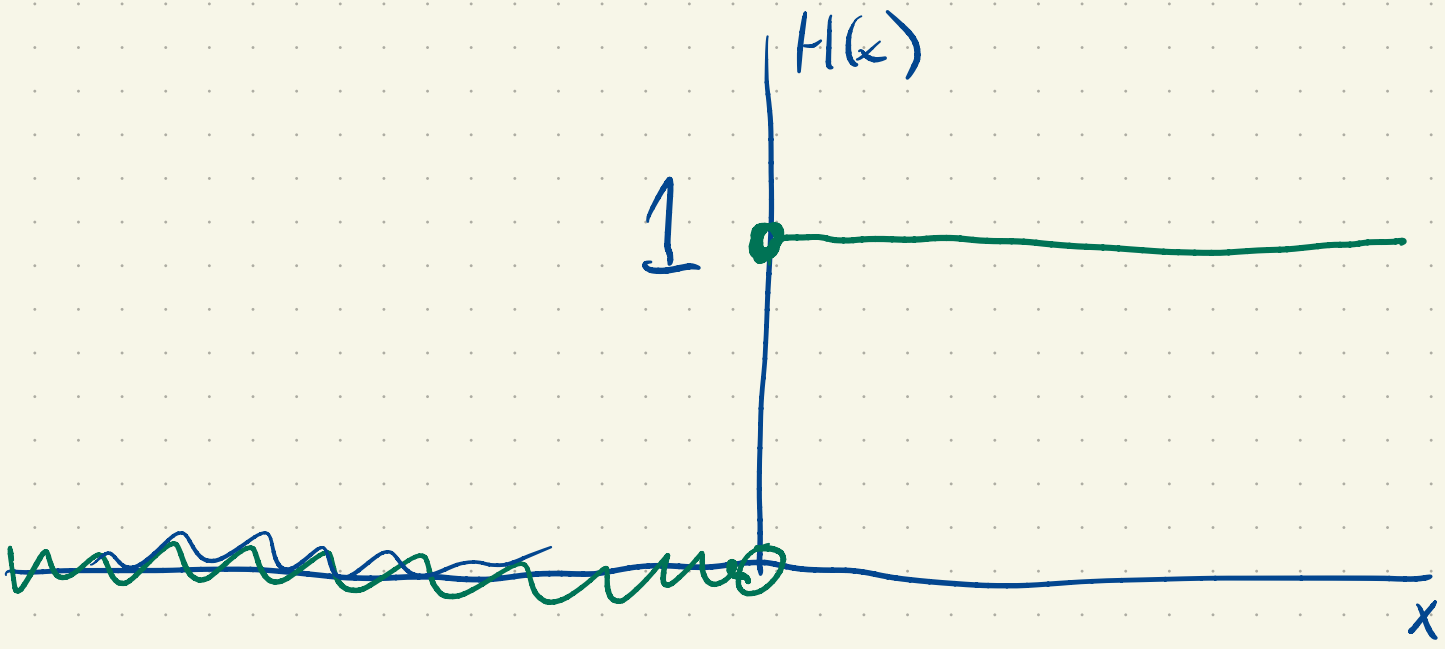
$f(x) = \frac{1}{x}$



$$\lim_{x \to 0} \frac{1}{x} \quad DNE$$

$$\lim_{x \to 0^+} \frac{1}{x} = +\infty$$

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$



Heaviside function $H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$

$\lim_{x \to 0} H(x)$ DNE

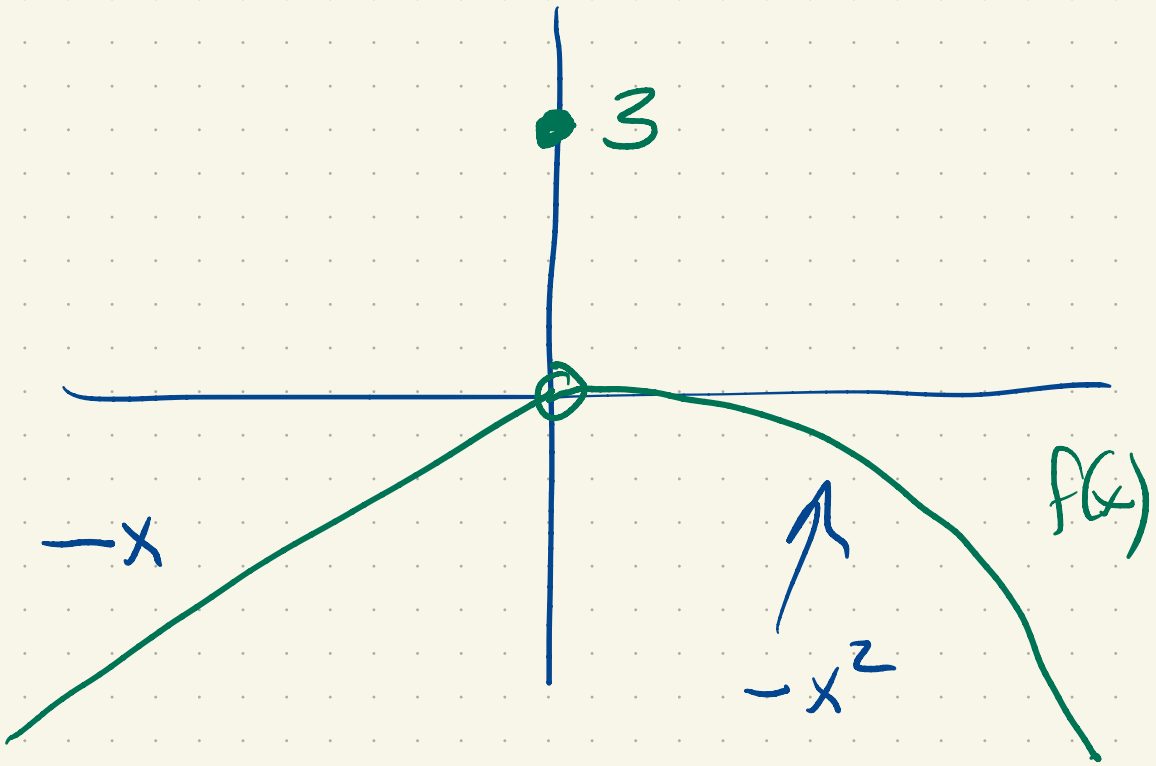
$\lim_{x \to 0^+} H(x) = 1$

$\lim_{x \to 0^-} H(x) = 0$

$$\lim_{x \to 0^+} \frac{-7}{x}$$

As $x \to 0^+$ x is small and positive

$$\lim_{x \to 0^+} \frac{-7}{x} = \frac{-7}{0^+} = -\infty$$



$f(0) = 3$

$\lim_{x \to 0} f(x)$

$\lim_{x \to 0^+} f(x) = 0$

$\lim_{x \to 0^-} f(x) = 0$