1/21

Average speed.

$d(t)$ $\quad$ d: distance traveled in miles

t: minutes

$d(0) = 0$

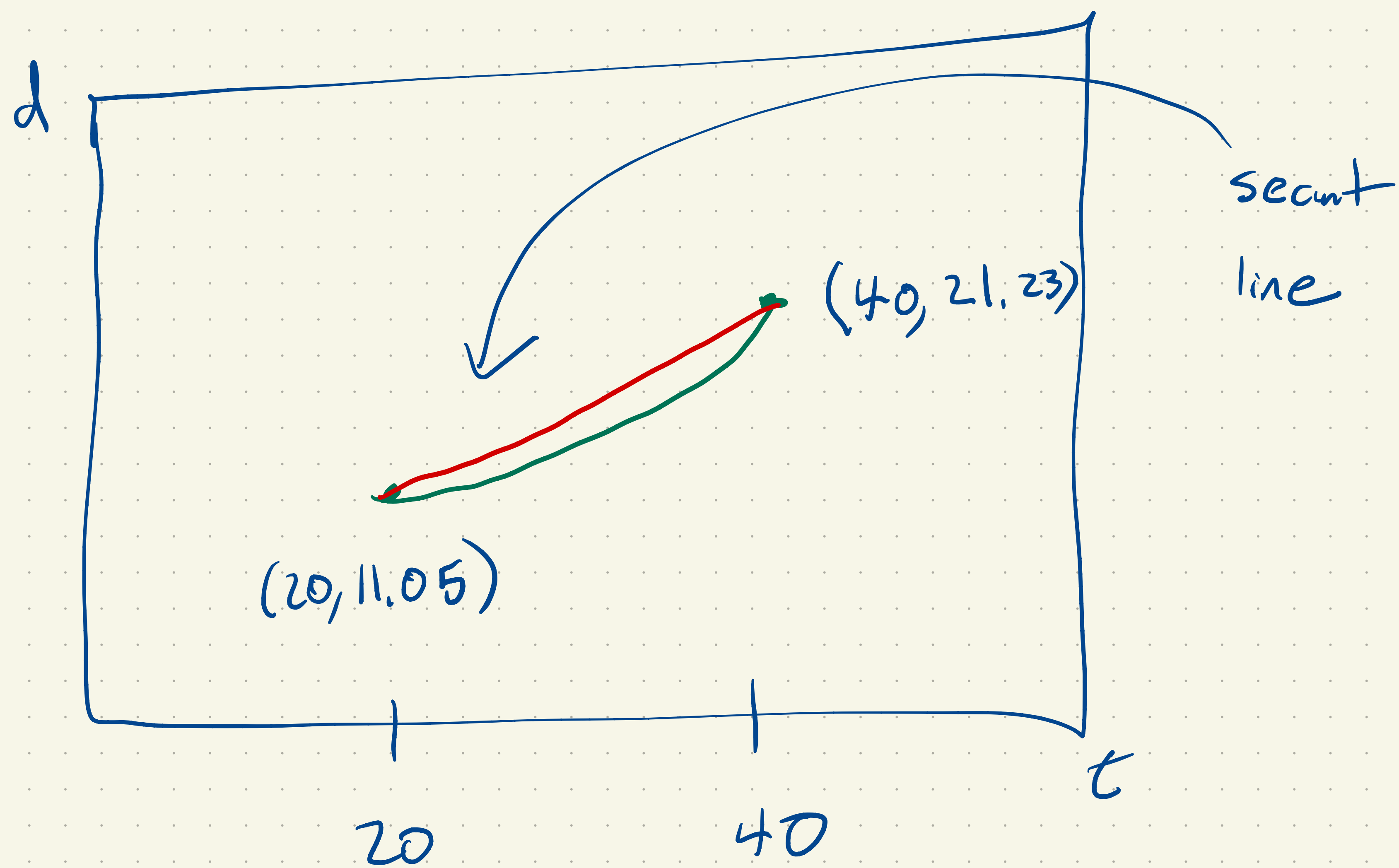
$d(5) = 4.8\ldots$

Distance traveled from time $t = t_0$ to $t = t_1$

$$\Delta d = d(t_1) - d(t_0)$$

$$\Delta t = t_1 - t_0$$

Average speed: $\dfrac{d(t_1) - d(t_0)}{t_1 - t_0}$



Average speed: $\dfrac{21.23 - 11.05}{40 - 20} = 0.51 \ \dfrac{\text{miles}}{\text{minute}}$

Rise: $21.23 - 11.05$

Run: 20

slope: $\dfrac{21.23-11.05}{20} = 0.51$

Average rates of change correspond to slopes of secant line.

Variation: $[t_0, t_0+h]$

length h

Average speed $\dfrac{d(t_0+h) - d(t_0)}{h}$

we can look at this for our favorite choices of h.

$$\frac{d(41+h) - d(41)}{h} \leftarrow s(h)$$

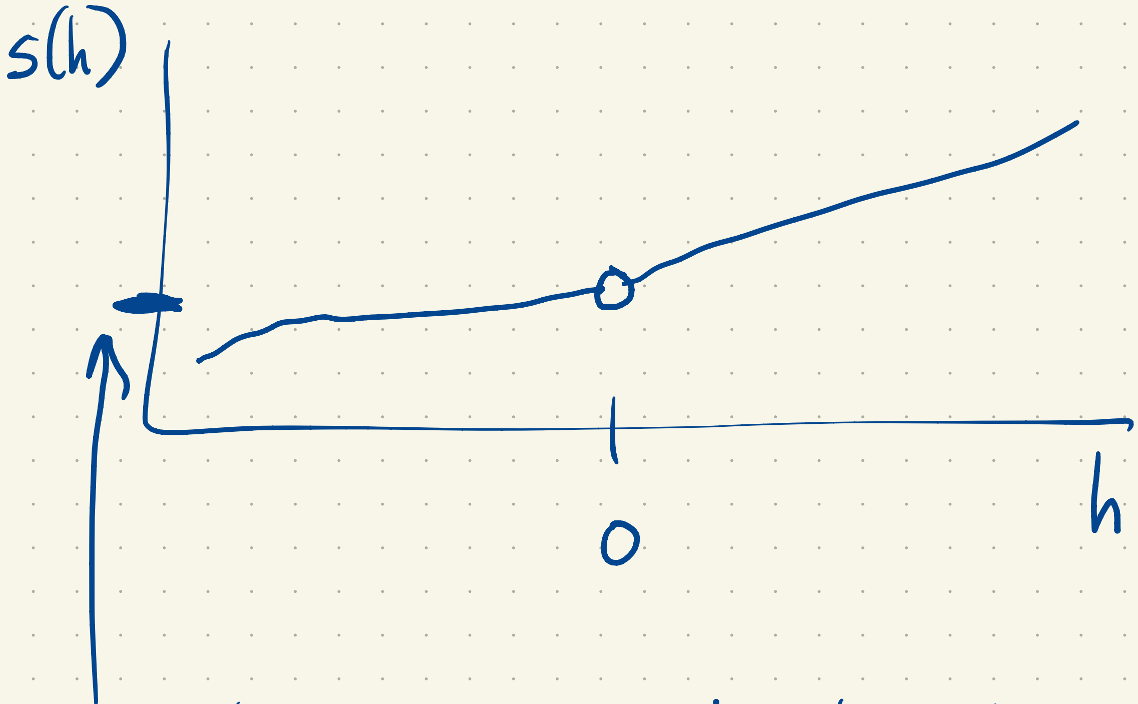
$h = 0$: $\frac{0}{0} \rightsquigarrow$ undefined

We can ask what happens as $h \to 0$

but we can't plug in $h = 0$.

$\to$ $s(1)$ average speed

$t_0 = 41$ to $t_1 = 42$



instantaneous speed at $t = 41$

$\sin(0) = 0$

$$f(x) = \frac{\sin(x)}{x}$$

$$f(0) = \frac{0}{0} \to \text{undefined.}$$

What happens as $x \to 0$.

x	$\sin(x)/x$
1	0.841...
0.1	0.9983...
0.01	0.99998...
0.001	0.9999998...

$= \sin(1)/1$

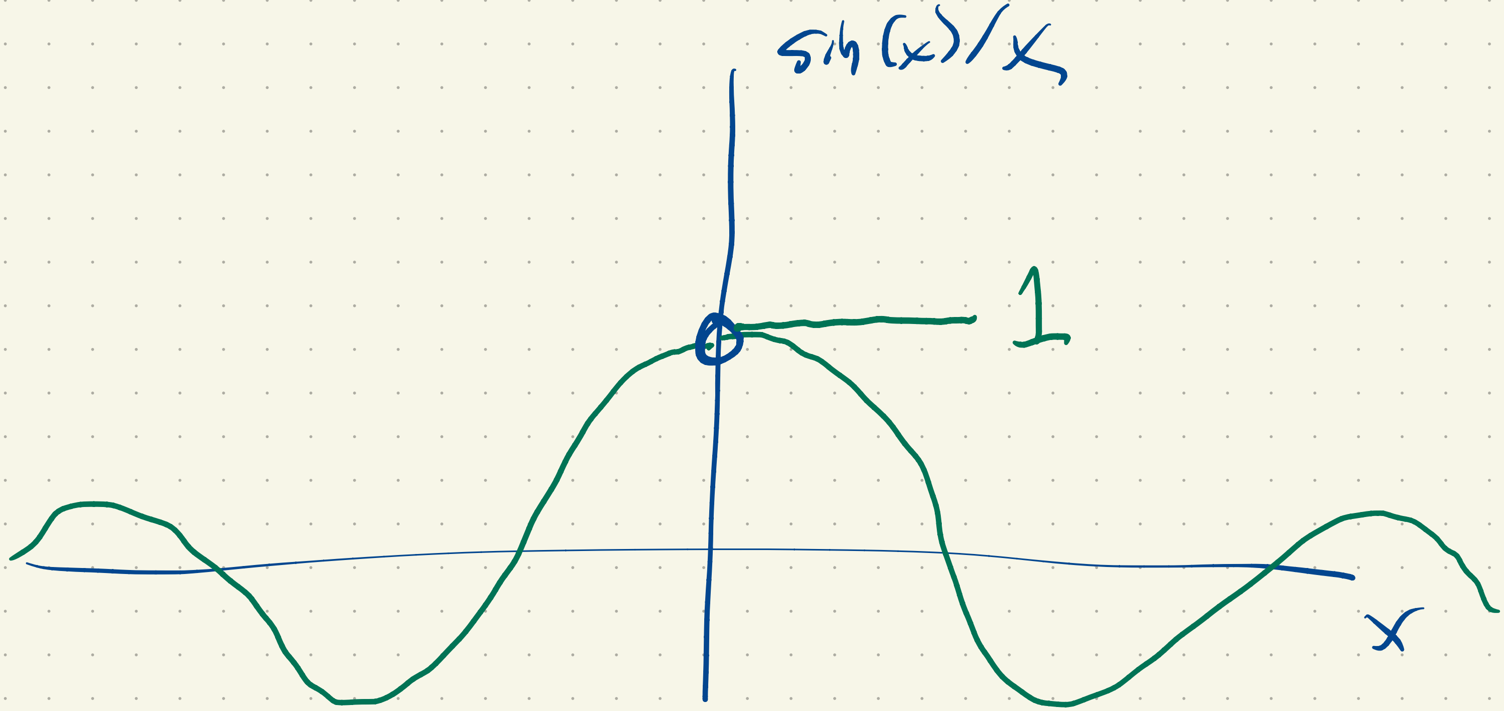
$= \sin(0.1)/0.1$

As $x \to 0$, $\frac{\sin(x)}{x} \to 1$

$$\boxed{\lim_{x \to 0} \frac{\sin(x)}{x} = 1}$$

$\frac{\sin(0)}{0} \leftarrow$ no-no

$\lim_{x \to a} f(x) = L$ if the values of $f(x)$ get closer and closer to L as x gets closer and closer to a.



average speed from
$t=41$ to $t=41+h$

$$\underbrace{\lim_{h \to 0} \frac{d(41+h)-d(41)}{h}}$$

instantaneous speed
at $t=41$

The average speeds approach the instantaneous speed as $h \to 0$.

Caribou $P(t) = 1000\,(1.1)^t$

$$P(0) = 1000 \cdot (1.1)^0 = 1000$$

animals

t is in years

$$P(1) = 1100$$

change in population from $t=0$ to $t=1$

$$P(1) - P(0) = 1100 - 1000 = 100$$

animals

$\underbrace{\text{Average rate of change}}_{\text{animals/year}}$ of population from $t=0$ to $t=1$?

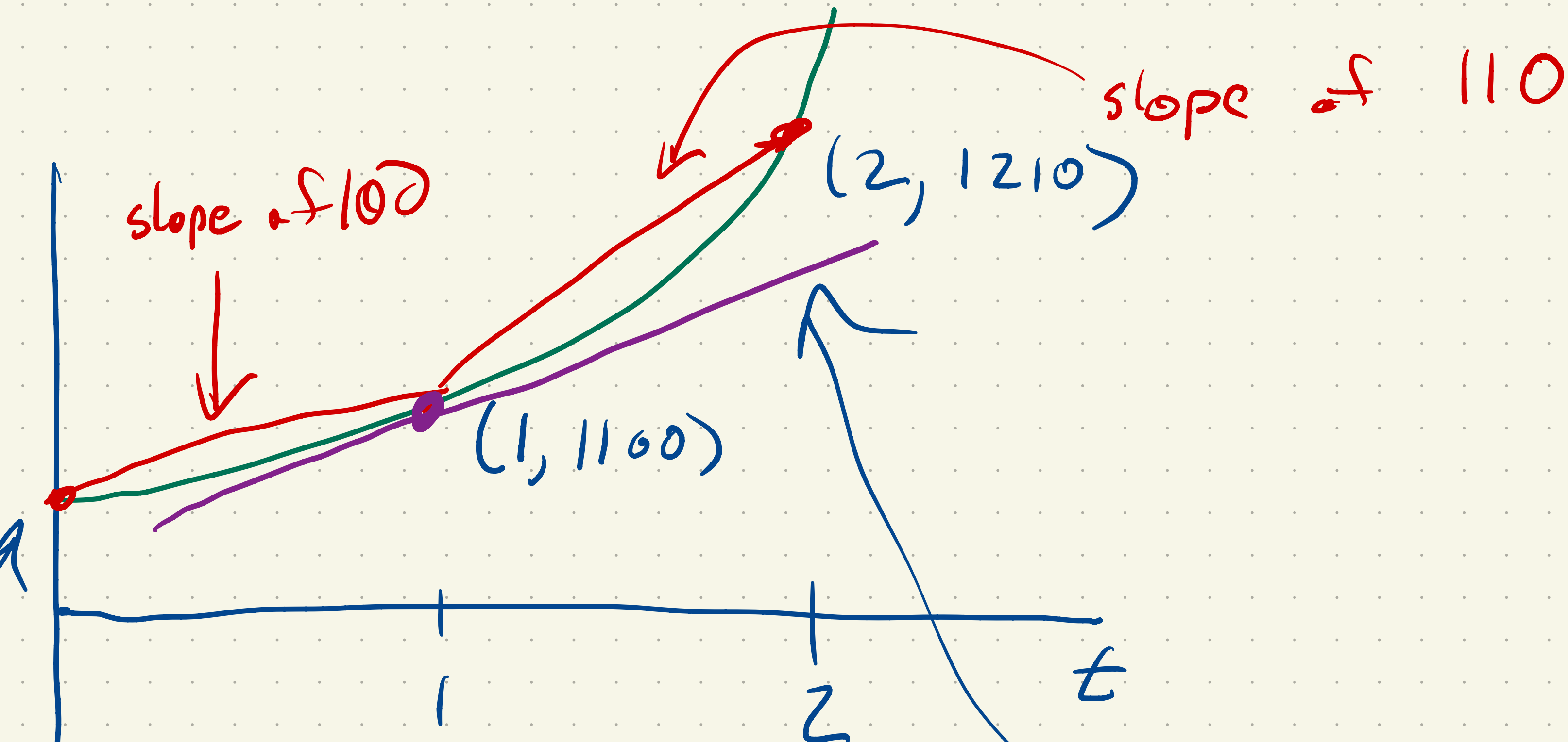
100 animals / year.

Average rate of change of population from $t=1$ to $t=2$

$$\frac{P(2)-P(1)}{2-1} = \frac{1000(1.1)^2 - 1000(1.1)^1}{2-1}$$

$$= 1000 \cdot 1.1 \; \frac{1.1 - 1}{1}$$

$= 110 \rightarrow \frac{\text{animals}}{\text{year}}$



(0, 1000)

tangent line slope tells you an instantaneous rate of change

How fast is the population changing right at $t=1$ year

$t=1$ $t=2.7$

$t=1$ to $t=1+h$

change in animals: $P(1+h) - P(1)$

length of time interval: h

$\underbrace{\text{average rate of change}}$ from $t=1$ to $t=1+h$

$\rightarrow \dfrac{\text{animals}}{\text{year}}$

$$\frac{P(1+h)-P(1)}{h}$$

to get the instantaneous rate of change

we look at $\lim_{h \to 0} \boxed{\dfrac{P(1+h) - P(1)}{h}} \to r(h)$

$h = 0 \Rightarrow \frac{0}{0}$

h	r(h)
0.1	105.34
0.01	104.89
0.001	104.84

approx 104.8

animals / year

$1000 \cdot \ln(1.1)$