Exponential functions $\quad C r^{at}$

Caribou:

$$p(t) = 1000(1.1)^t$$

claim: by a good choice of a

$$1000(1.1)^t = 1000 \cdot 2^{at}$$

This population then doubles every time

t goes up by? $\frac{1}{a}$

$2^{a \cdot 0} = 1$ $\qquad$ $2^{at} = 2$

$at = 1 \qquad t = 1/a$

$2^{a\left(\frac{2}{a}\right)} = 4$ $\qquad$ $0 \quad \frac{1}{a} \quad \frac{2}{a}$

$2^{at} \quad 1 \rightarrow 2 \rightarrow 4$

To rewrite we need logarithms

Recall $\log_{10}(10^x) = x \qquad 10^{\log_{10} y} = y$

$$\log_e(e^x) = x \qquad e^{\log_e y} = y$$

$e \approx 2.7$

$$\log_7(7^x) = x \qquad 7^{\log_7 y} = y$$

This is an example of an inverse function.

$$f^{-1}(f(x)) = x \qquad f(f^{-1}(y)) = y$$

Log rules

$$10^{x+y} = 10^x 10^y \longleftrightarrow \log_{10}(ab) = \log_{10} a + \log_{10} b$$

$$(10^x)^y = 10^{xy} \longleftrightarrow \log_{10}(a^b) = b \log_{10}(a)$$

$$10^0 = 1 \longleftrightarrow \log_{10}(1) = 0$$

These rules apply to all bases ($10 \rightarrow e$)

Common log: $\log_{10}$

Natural log $\underbrace{\log_e}_{\ln}$

$\ln$

log → science, eng. $\log_{10}$

log → math, computers, Matlab $\log_e$

$$1000\,(1.1)^t = 1000\ 2^{at}$$

$$(1.1)^t = 2^{at}$$

$$\log_{10}\left((1.1)^t\right) = \log_{10}\left(2^{at}\right)$$

$$t \log_{10}(1.1) = at \log_{10}(2)$$

$$\log_{10}(1.1) = a \log_{10}(2)$$

$$a = \frac{\log_{10}(1.1)}{\log_{10}(2)} \simeq 0.1375$$

Time to double: $\frac{1}{a} \approx 7.27$ (years)

$\log_{10}$ and $\log_e$ are [illegible]

$$\log_{10}(10^y) = y \; \cancel{\log_{10}(10)}$$

$$\log_e(10^y) = y \log_e(10)$$

$$y = \frac{\log_e(10^y)}{\log_e(10)} \qquad y = \log_{10}(10^y)$$

$$\log_{10}(10^y) = \frac{\log_e(10^y)}{\log_e(10)} \qquad x = 10^y$$

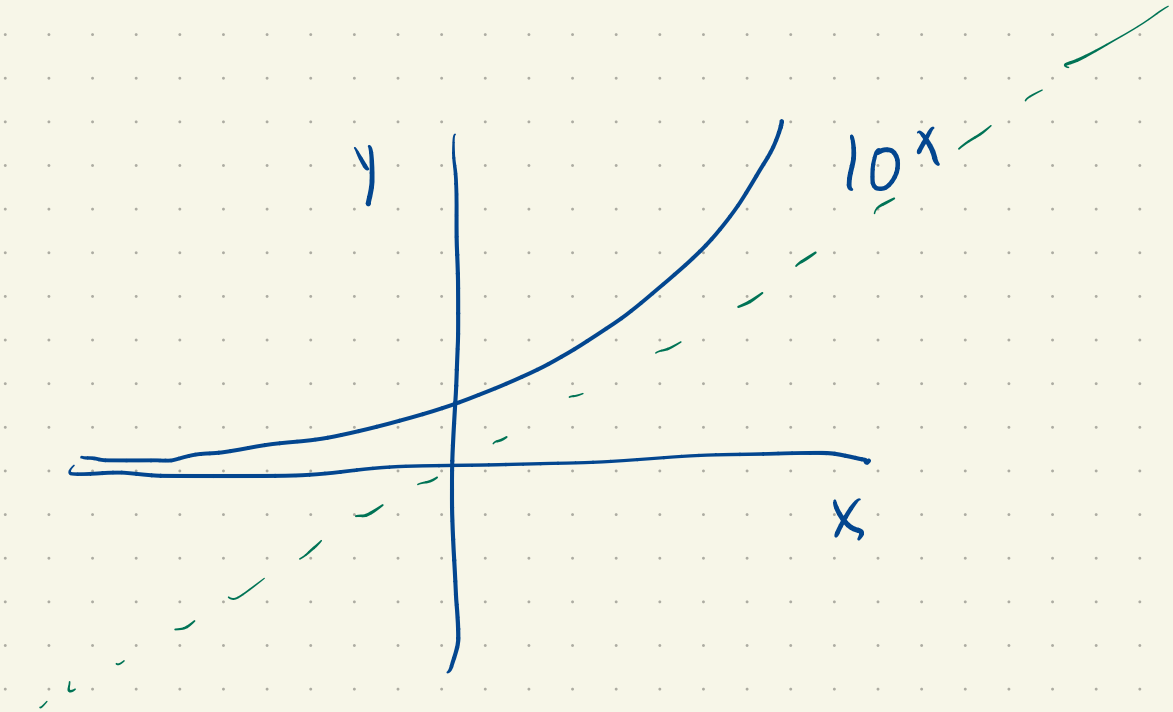
$$\log_{10}(x) = \frac{\log_e(x)}{\log_e(10)}$$

$$= \frac{\ln(x)}{\ln(10)}$$

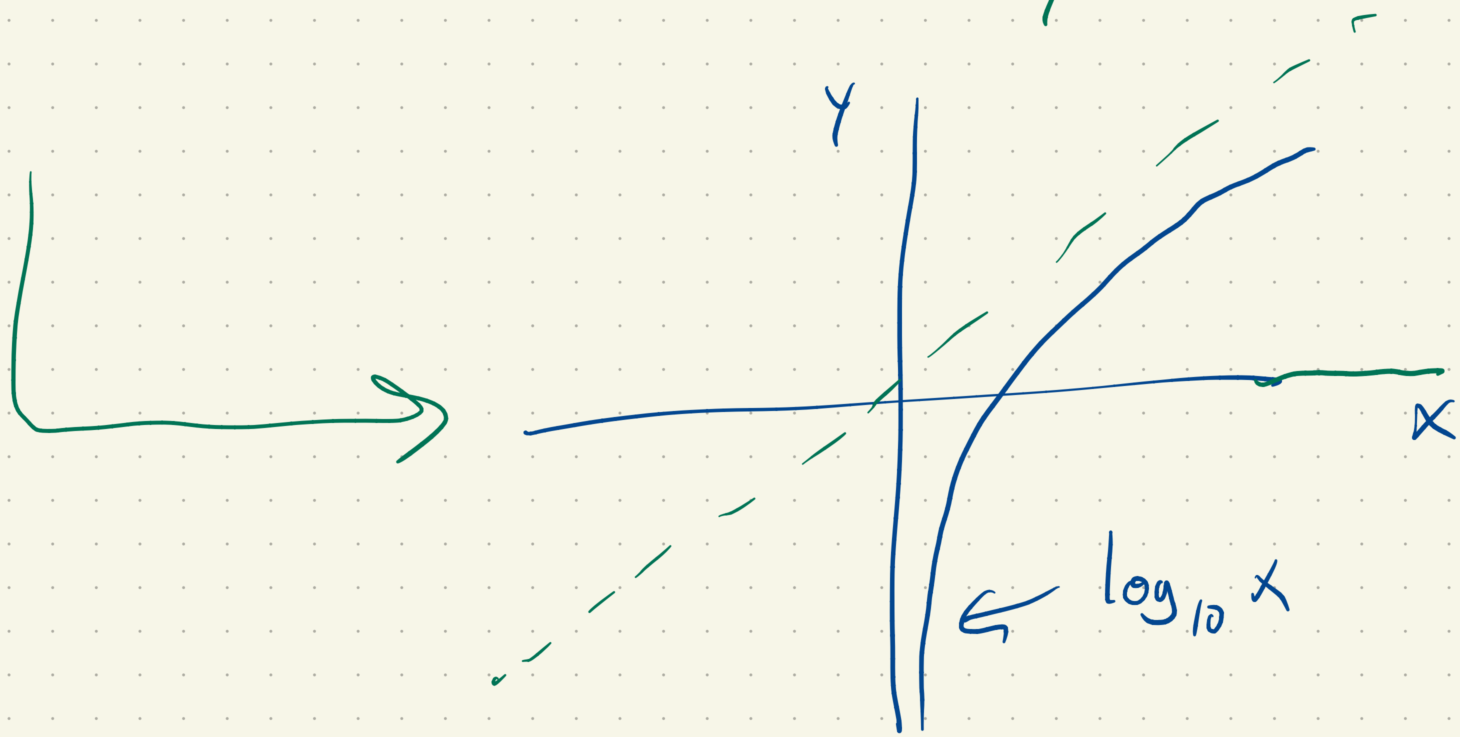
$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

change of base

How do we graph $\log_{10}(x)$?



For an inverse function, like $\log_{10}$, we swap x, y



$\log_{10}(x)$ is only defined for $x > 0$