

The in-class midterm will consist of easy computations, statements of definitions, and straightforward proofs. The idea is to encourage you to go back and review all the things we've learned up to now. I've listed some study ideas below. Not everything on the exam is necessarily on this list

- Know Taylor's Theorem with remainder. Be able to apply it to estimate the error of a Taylor polynomial.
- Be able to estimate the truncation error for a finite difference approximation of a derivative.
- Be able to compute the order of accuracy of a finite difference scheme.
- Be able to implement a basic numerical method in pseudo code. Newton's method, forward Euler for ODEs or the heat equation, backwards Euler for ODEs or the heat equation would all be reasonable algorithms.
- What does consistency mean? Convergence?
- Be able to compute the characteristic polynomial for zero stability for a linear multi-step method. What is zero stability? Why is zero stability important?
- Be able to compute the characteristic polynomial for absolute stability for a linear multi-step method. What is the region of absolute stability for an approximation scheme for an ODE. Why do we care about it?
- What is an A-stable method? Why care about these?
- How do you compute the region of absolute stability for a Runge Kutta method?
- Given a linear system of ODEs $u' = Au$ with initial condition $u(0) = u_0$, describe a method for finding a solution (You may assume that A admits a basis of eigenvectors).
- What is the maximum principle for the heat equation?
- What are the θ methods for solving the heat equation?
- Why does absolute stability play a critical role in analysing convergence of finite difference methods for the heat equation. That is, why do you expect when applying the forward Euler method that there will be a restriction on the time step if you refine the space grid sufficiently. Why do you expect that this relationship will be $k \sim h^2$? Your answer should involve eigenvalues of ∂_x^2 .
- Why is backward Euler preferable to forward Euler for solving the heat equation? What advantages does Crank-Nicholson possess?