**1.** Consider the matrix

$$A = \begin{pmatrix} 23 & -8 & 4 \\ 21 & -8 & 5 \\ -126 & 42 & -19 \end{pmatrix}.$$

a) Show that $v_1 = [-1, -2, 3]$, $v_2 = [1, 3, 0]$ and $v_3 = [0, 1, 2]$ are eigenvectors of $A$, and determine their associated eigenvalues.

b) Compute the solution of

$$u' = Au$$

with initial condition $u(0) = v_3$. Show, by plugging your solution into the ODE, that your solution really is a solution.

c) Compute the solution of

$$u' = Au$$

with initial condition $u(0) = v_2 + v_3$. Show, by plugging your solution into the ODE, that your solution really is a solution.

d) Determine the exact solution of

$$u' = Au$$

with initial condition $u(0) = [1, 5, 5]$.

**2.** Suppose you wish to apply the RK4 method to solve the ODE of the previous problem. What is the largest time step you can use before issues concerning absolute stability arise in your solution?

**3.**

a) Use your Newton solver from last week's homework to implement the trapezoidal rule for solving systems of ODEs.

b) Determine the exact solution to the problem

$$\begin{aligned} u' &= 1 \\ v' &= v - u^2 \end{aligned} \tag{1}$$

with initial condition $u(0) = 0$ and $v(0) = 1$.

c) Test your solver against the previous exact solution and confirm that it has the predicted order of accuracy.

**4.** Consider this one-step (Runge-Kutta) method, the implicit midpoint method,

$$\begin{aligned} u_* &= u_n + \frac{h}{2} f(t_n + h/2, u_*) \\ u_{n+1} &= u_n + h f(t_n + h/2, u_*) \end{aligned} \tag{2}$$

The first equation (stage) is Backward Euler to determine an approximation to the value at the midpoint in time and the second stage is the midpoint method using this value.

a) Determine the order of accuracy of this method.

b) Determine the stability region.

c) Is this method A-stable?