Let A be a subset of $\mathbb{R}^n$. A map $f : A \to \mathbb{R}^m$ if whenever $x_j \to x$ in A, $f(x_j) \to f(x)$ in $\mathbb{R}^m$.

It is easy to show (you need not do this) that such a map is continuous if and only if each component function $f_k : A \to \mathbb{R}, 1 \le k \le m$ is continuous.

A path in a set $B \subseteq \mathbb{R}^m$ is a continuous map $\gamma : [0,1] \to B$. Given b_0 and b_1 in B, a path from b_0 to b_1 is a path γ in B with $\gamma(0) = b_0$ and $\gamma(1) = b_1$. The set is path connected if for all b_0 and b_1 in B, there is a path from b_0 to b_1.

1. By a concrete construction, show that $S^1 \subseteq \mathbb{R}^2$ is path connected.

2. Use the intermediate value theorem to show that $\mathbb{R} \setminus \{0\}$ is not path connected.

3. Show that $SU(2)$, the set of unit quaternions, is path connected. You may use the fact that a set B is path connected if and only if there exists an $\hat{b} \in B$ such that for any b in B there is a path from $\hat{b}$ to b.

4. Breifly explain why matrix multiplication is a continuous map from $R^{n\times n} \times R^{n\times n}$ to $R^{n\times n}$.

5. 3.2.1-3.2.3. You are encouraged to have a brief solution; full rigor is not required.