

The in-class final will consist of easy computations, statements of definitions, and straightforward proofs. The idea is to encourage you to go back and review all the things we've learned up to now. I've listed some study ideas below. Not everything on the exam is necessarily on this list, and the list certainly too long to have every topic included in a two-hour exam.

- What is a matrix Lie group? What is a Lie algebra? What is a matrix Lie algebra?
- What is a tangent vector to a matrix Lie group?
- How do you represent rotations in \mathbb{R}^4 and \mathbb{R}^3 using quaternions?
- What is the relationship between $SU(2)$ and $SO(4)$? What is the relationship between $SU(2)$ and $SO(3)$? You should know the details here.
- How do you represent complex numbers as matrices? Quaternions?
- Why is multiplication by a unit quaternion an isometry?
- How are the groups $O(n)$, $SO(n)$, $U(n)$, $SU(n)$, and $Sp(n)$ defined? What is the common theme connecting these?
- What are the Lie algebras of the groups in the item above?
- Know the matrix exponential. How is it defined? When is $\exp(A+B) = \exp(A)\exp(B)$? Why is it that if $A \in \mathfrak{o}(n)$ that $\exp(A) \in O(n)$?
- What is an ideal of a Lie algebra?
- What is a normal subgroup of a group? What is the quotient group G/N when N is a normal subgroup of G ? What is a simple group? What is a simple Lie algebra?
- How do represent reflections using quaternions?
- What does it mean for a matrix group to be path connected? Why is $O(n)$ not path connected?
- What is a maximal torus? What is the center of a group? What are the centers of the groups $O(n)$, $SO(n)$, $U(n)$, $SU(n)$, and $Sp(n)$? Why is the center a normal subgroup?
- How is $\mathfrak{gl}(n, \mathbb{C})$ related to $\mathfrak{u}(n)$?