

1. 5.15
2. 5.16 (Read Def. 5.45 first)
3. 5.17
4. 5.18
5. 5.19

If you have not seen the definition of quotient of vector space by a subspace, it's not a hard concept. Given a vector space  $X$  and a subspace  $W$  we say  $x \sim z$  if  $x - z \in W$ , or alternatively if  $x = z + w$  for some  $w \in W$ . It's easy to see that this is an equivalence relation and we write the equivalence class of  $x$  as  $x + W$  rather than  $[x]$ . We define  $(x + W) + (z + W)$  by  $(x + z) + W$  and  $\alpha(x + W) = \alpha x + W$ . You should show that these operations are well defined as a preamble to starting 5.19. With these operations, the set of equivalence classes, written  $X/W$ , is a vector space, though you do not need to prove this.

6. 5.22 (We sketched this in class; write the details down.)