1. R & Y 3.21

2. R & Y 3.22

3. R & Y 3.23

4. R & Y 4.3

5. Consider the map $I : C[0,1] \to C[0,1]$ given by

$$(I(f))(x) = \int_0^x f(s)\, ds.$$

Find a sequence of functions f_n such that $\|f_n\|_\infty = 1$ and such that $\|If_n\|_\infty \to 0$.

Then tell me what this has to do with Example 4.10.

6. For each pair p and q with $1 \le p, q \le \infty$ determine whether the map $z \mapsto z$ from (Z, ℓ^p) to (Z, ℓ^q) is continuous.

7. Consider Z with the ℓ^2 norm. Fix $w \in \ell^p$ for some $1 \le p \le \infty$ and define

$$T(z) = \sum_{k=1}^{\infty} w_k z_k.$$

Determine the values of p such that T is necessarily continuous.

8. [Extra Credit] For each pair p and q with $1 \le p, q \le \infty$ determine whether the map $f \mapsto f$ from $(C[0,1], L^p)$ to $(C[0,1], L^q)$ is continuous.