1. Prove that ℓ^∞ is complete.

2. Consider the norms $||\cdot||_1$, $||\cdot||_2$, and $||\cdot||_\infty$, on $\mathbb{R}^n$. Since all norms on $\mathbb{R}^2$ are equivalent, there are constants m and M such that

$$m||x||_1 \le ||x||_2 \le M||x||_1$$

for all $x \in \mathbb{R}^2$. Find, with proof, the best such constants and make a diagram that illustrates this fact.

Now repeat this exercise for the remaining two pairs of norms (i.e. the pair $||\cdot||_1$ and $||\cdot||_\infty$ and the pair $||\cdot||_2$ and $||\cdot||_\infty$).

3. Show that if X is a Banach space and $S \subseteq X$ is a closed subspace, then S is complete (and hence a Banach space).

4. R & Y 2.10

5. R & Y 2.11(b)

6. R & Y 2.12

7. R & Y 2.13

8. R & Y 2.14