1. R & Y 2.1

They mean here, show that the putative norm in Example 2.8 really is a norm.

2. R & Y 2.2

For part b), I know that we have not discussed the Lebesgue space $L^1[0,1]$. Don't fret. Up to a white lie, $L^1[0,1]$ consists of functions mapping $[0,1]$ to $\mathbb{R}$, and includes the continuous functions. For a continous (and hence Riemann integrable function) f,

$$||f||_{L^1[0,1]} = \int_0^1 |f|.$$

That's enough background to solve the problem.

3. R & Y 2.3

4. R & Y 2.4

5. R & Y 2.5

6. Let X be an normed vector space. Show that a sequence $\{x_n\}$ in X converges to x if and only if

$$\lim_{n\to\infty} ||x_n - x|| = 0.$$

7. Suppose f and g are continuous functions on $[a,b]$. Show that

$$\int_a^b fg \le \left[\int_a^b f^2\right]^{\frac{1}{2}} \left[\int_a^b g^2\right]^{\frac{1}{2}}.$$

Then show that if f and g are continuous on $[a,b]$,

$$||f+g||_2 \le ||f||_2 + ||g||_2$$

where

$$||f||_2 = \left[\int_a^b f^2\right]^{\frac{1}{2}}.$$

8. R & Y 2.6