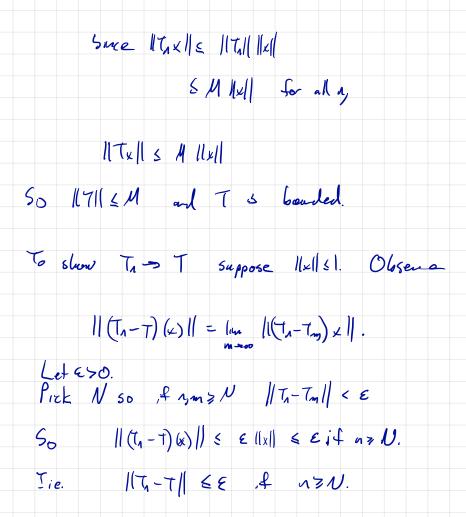
Then: II Y is Barndy B(4,2), with opentor nom, is a Barch space.

Pf: Let 27,3 be Caudy in B(X,Y). Let XE X. $\| T_n \times - T_m \times \| = \| (T_n - T_m) (\infty) \|$ 5 || t- tm || ||x|| So 2 T1 x 3 is Cauchy in 4 and conveges to a limit Tx. I claim that T is liker and Tr > T. For $x_1 \ge e X$, $T(x_1 \ge 1_{M} = 1_M T_M(x_1 \ge 1_M)$ - lim ちょちん こ = Tx+ TZ.

cle,

Recall Candy seques are bounded, so I M, 117,115M

Notre || Tx|| = by || Tx x|



Prop: Suppose X is a normed linear space.

Then there exists a Barach space Y

and a map T: X- Y such this

1) ||Tx||= ||x||y & xeX

2) $TX = \gamma$

e.g. Z with l^2 norm, his l^2 as a completion, We often ursualize $X \in T$ by r.d. with TX(D such a map is called an isometry. It preserves distance Pf: Let Y be the set of Candy sequences in Y. I will identify two country sequences (x1, ...) (z., ...)

A (K, 2, --..) is still Coundry, and write (x,) ~ (x,)

Bo-ty: (xn)~ (xn) (xn) 2641) -> (xn) ~64)

(41) 2(41), (41) 2(21) = (4)~ (21) Y: sol of equily classes.

Exercise: Y is a vector space

Note [11x11] - 11x101 [SIX - x11]

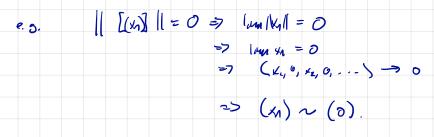
(am IIXA/I. exists. 5.

```
 \sum_{x \in cise:} :f(x_n) \wedge (z_n), ||_{x_n} ||_{(x_n)} ||_{=} ||(z_n)||.
```

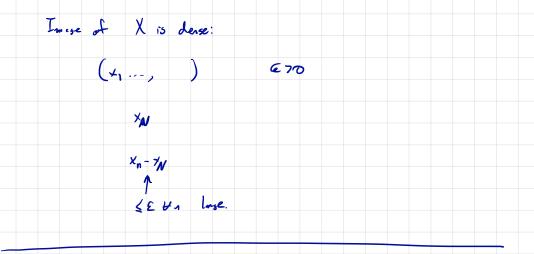
lam.

```
|[[(x1)]] is low [|x1|].
```

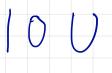
```
Exorise: [] ·] is a norm on 1.
```



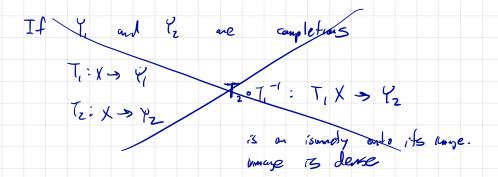
M-p Cun X to Y: X-> (x). Clearly an isomoty





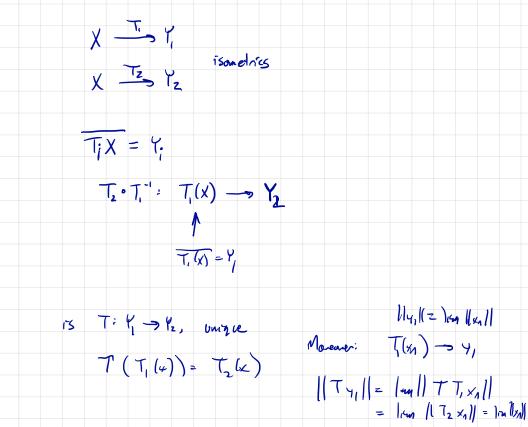






The: Suppose X is a hornal spice Y is a Bauch space, and suppose W is dense in X. Given $S \in B(W, Y) = \overline{I}! T \in B(Y, Y) = T_W^{-5}$, and ||T|| = ||S||

Application: Completions are essentially unique.



Pf: Let x CX and pick war -> x. Observe (115 mn - 5 mm) = 115 (m - m) 6 11511 11 wu-wm/ 50 Il Swall is caudy and conversos to a lawit y Morecover, , f in -> × they $w_n - w_n \rightarrow 0$ S(w, - in) - 0 by confirmedy. Sun -y Sin -> ? . S(un - in) - y - y => ノ=シ. Veletue Tx = lum Sun. Need to verty a) T is linear 6) T is bounded. c) 11711 = 1151

a: Gwen x, 5 wasx in st $T(x+\hat{x}) = \lim S(u_x + \hat{u_x})$ = (m Swn + lun Sws $= T_X + T_S$ ditto for scalar mult. b) $||T_x|| = \lim_{x \to \infty} ||Sw_n||$ But 115 m, 11≤ 11511 /1m, 11 → 11511 1(x) (. δ_{0} $||T_{x}|| \leq ||S|| ||x||. S_{0} ||T|| \leq ||S||.$ c) Execise: 11711311511. Finally, if T' is another such given X e X, T'(x) = lun T'(un) = lun S(un) - Tx. So T is unique.