In the first part of this worksheet we will get to know a method for computing an approximation of $\sqrt{2}$ to many digits of accuracy using only addition, subtraction, multiplication and division, and indeed using only a few such operations.

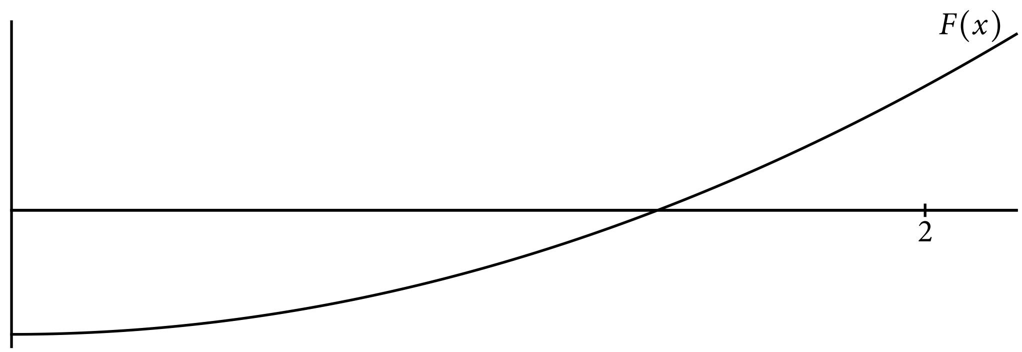
**1.** Consider the function

$$F(x) = x^2 - 2.$$

If we solve $F(a) = 0$ for some $a \geq 0$, what is the value of $a$?

**2.** Find the linearization $L(x)$of $F(x)$ at $x = 2$. Leave your answer in point-slope form.

**3.** I've graphed $F(x)$ for you below. Add to this diagram the graph of $L(x)$.




**4.** Find the number $x_1$ such that $L(x_1) = 0$.

**5.** What good is the number $x_1$? Keep in mind that you want to solve $F(x) = 0$. You solved $L(x) = 0$ instead.

**6.** In the diagram above, label the point $x_1$ on the $x$-axis.

**7.** Let's do it again! Find the linearization $L(x)$ of $F(x)$ at $x = x_1$.

**8.** Add the graph of this new linearization to your diagram on the first page.

**9.** Find the number $x_2$ such that $L(x_2) = 0$. Then label the point $x = x_2$ in the diagram.

**10.** To how many digits does $x_2$ agree with $\sqrt{2}$

**11.** Let's be a little more systematic. Suppose we have an estimate $x_k$ for $\sqrt{2}$.

- Compute $F(x_k)$.
- Compute $F'(x_k)$.
- Compute the linearization of $F(x)$ at $x = x_k$.

  $L(x) =$

- Find the number $x_{k+1}$ such that $L(x_{k+1}) = 0$. You should try to find as simple an expression as you can.

**12.** Starting with $x_0 = 2$, compute $x_1$ and $x_2$ with your shiny new formula. Verify that they agree with your earlier expressions for $x_1$ and $x_2$.

**13.** Compute $x_4$. To how many digits does it agree with $\sqrt{2}$?

## Newton's Method In General

We wish to solve $F(x) = 0$ for a differentiable function $F(x)$. We have an initial estimate $x_0$ for the solution.

**14.** Try to solve

$$e^{-x} - x = 0$$

by hand.

**15.** Explain why there is a solution between $x = 0$ and $x = 1$.

**16.** Starting with $x_0 = 1$, find an approximation of the solution of $e^{-x} - x = 0$ to 6 decimal places. During your computation, keep track of each $x_k$ to at least 10 decimal places of accuracy.