Consider the function $f(x) = x^2$ on the interval $[-1, 3]$

1. Find the slope of the secant line of the graph of $f(x)$ from $x = -1$ to $x = 3$.

2. Find a value of x in $[-1, 3]$ where $f'(x)$ equals the value you found in problem 1.

3. Make a sketch of the graph of $f(x)$ and add to it the secant line from problem 1 and the tangent line at the location found in problem 2. What property do the secant line and tangent line have?

4. Repeat the exercise of problems 1-3 with $g(x) = 1/x$ on [1,5].

5. Repeat the exercise of problems 1-3 with $\sin(x)$ on $[0, 2\pi]$.

Mean Value Theorem. If f is a continuous function on an interval $[a, b]$ that has a derivative at every point in (a, b), then there is a point c in (a, b) where

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

6. What is the geometric meaning of the value $\frac{f(b)-f(a)}{b-a}$?

7. Consider the function $f(x) = |x|$ on [-1,1]. The Mean Value Theorem would say that there is a c in $(-1, 1)$ where

$$f'(c) = \frac{|-1| - |1|}{1 - (-1)} = 0.$$

Is there such a point? Why doesn't this violate the Mean Value Theorem?

Rolle's Lemma (Baby Mean Value Theorem). If f is a continuous function on an interval $[a, b]$ that has a derivative at every point in (a, b), and if $f(a) = f(b)$, then there is a point c in (a, b) where

$$f'(c) = 0.$$

8. Why is this a special case of the Mean Value Theorem?

9. Draw a picture that illustrates Rolle's Lemma.

Proof of Rolle's Lemma:

10. Suppose f is a continuous function on $[a, b]$ and $f'(x) \geq 0$ for every x in (a, b). How do $f(a)$ and $f(b)$ compare?

11. Suppose f is a continuous function on $[a,b]$ and $f'(x) \leq 0$ for every x in (a,b). How do $f(a)$ and $f(b)$ compare?

12. Suppose f is a continuous function on $[a,b]$ and $f'(x) = 0$ for every x in (a,b). How do $f(a)$ and $f(b)$ compare?

13. Suppose on some interval (a,b) that $f(x) = C$ for some constant C. What can you say about $f'(x)$ on (a,b)?

14. Suppose $f'(x) = 0$ on an interval (a,b). Then there is a constant C such that $f(x) = C$ for all x in (a,b). Why?

15. Suppose $f'(x) = g'(x)$ on an interval (a, b). Then there is a constant C where $g(x) = f(x) + C$. Why?

16. Suppose a car is traveling down the road and in 30 minutes it travels 32.7 miles. What does the Mean Value Theorem have to say about this?

Proof of Mean Value Theorem