

There is a class of problems in calculus, known as related rate problems. Here's the idea. You know the rate of change (often with respect to time) of one quantity, such as the volume of a spherical balloon. You want to know the rate of change of some other related quantity (e.g. the radius of the balloon). Here are the steps you take to solve a problem like this:

1. Identify the quantity you already know a rate of change of (say,  $V$ , so you know  $dV/dt$ ).
2. Identify the quantity you want a rate of change of (say,  $r$ , so you want  $dr/dt$ ).
3. Find an equation that relates the two quantities ( $V$  and  $r$ ). This can be the hard part. Drawing a picture can help.
4. Now take a derivative with respect to  $t$  of both sides of the equation, treating both  $V$  and  $r$  as functions of  $t$ .
5. Substitute all known data into the result (typically  $V$ ,  $r$  and  $dV/dt$ ) to determine  $dr/dt$ .

We'll repeat this procedure with a bunch of examples.

1. Air is being pumped into a spherical balloon so that its volume increases at a rate of 4.5 ft<sup>3</sup>/min. How fast is the radius of the balloon increasing when the diameter is 4 ft?





6. The standard 12 foot ladder rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?
7. A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 mi north of the intersection and the car is 0.8 mi to the east, the police determine that the distance between them and the car they are chasing is increasing at a rate of 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car? [Hint: You'll need to relate *three* quantities here!]