Final Review – Last Day

Final Exam: Wednesday May 2 from 1:00 PM - 3:00 PM.

Section F01 (Faudree) Grue 208

Section F02 (Maxwell) Grue 206

Calculus Nutshell

1. limits
2. derivatives
3. integrals
4. How do you find/evaluate them and what do they tell you?

Chapter 5

1. (Warm-up) Evaluate.

   (a) $\displaystyle\int_0^{\pi/4} \frac{\sec^2 t}{\tan t + 1} dt = \ln\left(\left|\tan t + 1\right|\right)\Big]_0^{\pi/4} = \ln\left(\tan\left(\tfrac{\pi}{4}\right)+1\right) - \ln\left(\tan(0)+1\right)$

   $= \ln(2) - \ln(1) = \ln(2)$

   (b) $\displaystyle\int_1^4 \frac{x-2}{\sqrt{x}} dx = \int_1^4 \left(x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}\right) dx = \frac{2}{3}x^{\frac{3}{2}} - 4x^{\frac{1}{2}}\Big]_1^4$

   $= \left(\frac{2}{3} 4^{3/2} - 4(4)^{1/2}\right) - \left(\frac{2}{3} \cdot 1^{3/2} - 4(1)^{1/2}\right)$

   $= \frac{16}{3} - 8 - \frac{2}{3} + 4 = \frac{14}{3} - 4 = \frac{2}{3}$

(c) $\int \left(\sec x \tan x + \frac{2}{\sqrt{1-x^2}} \right) dx$ $= \sec x + 2 \arcsin x + C$

(d) $\int \frac{x}{(x-2)^3} dx$ $= \int x(x-2)^{-3} dx = \int (u+2)u^{-3} du = \int u^{-2} + 2u^{-3} du$

$u = x-2$

$du = 2x$

$x = u+2$

$= -u^{-1} - u^{-2} + C = -(x-2)^{-1} - (x-2)^{-2} + C$

2. A particle is moving with velocity $v(t) = 2t - 1/(1+t^2)$ measured in meters per second.

   (a) Find and interpret $v(0)$.

   $v(0) = 0 - 1 = -1$ m/s.

   The particle is moving to the left. (or its position is decreasing.)

   (b) Find the displacement for the particle from time $t = 0$ to time $t = 4$. Give units with your answer.

   $\int_0^4 v(t)\,dt = \int_0^4 2t - \frac{1}{1+t^2}\,dt = t^2 - \arctan(t) \Big]_0^4$

   $= 16 - \arctan(4) \approx 14.7$ m

   (c) If D is the *distance* the particle traveled over the interval $[0, 4]$, is D larger or smaller or exactly the same as your answer in part (b)? Justify your answer.

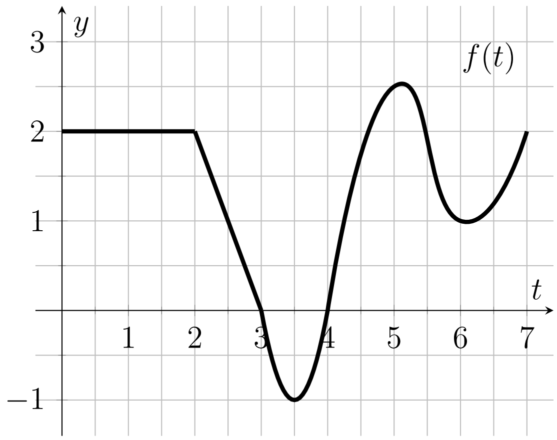
   $D < 14.7$.

   Since the velocity is initially negative but the displacement is positive, the particle must have moved left than right 14.7 meters past its starting position.

   (d) Assuming $s(0) = 1$, find the position of the particle.

   $1 + 14.7 = \underline{15.7 \text{ m}}$.

3. The graph of $y = f(t)$ is displayed below. A new function is defined as $H(x) = \int_0^x f(t)\,dt$.



(a) Find $f(3)$. 0

(b) Find $g(3)$ $4 + \frac{1}{2}(2 \cdot 1) = 5$

(c) Find all x-values for which $g'(x) = 0$. $x = 3, 4$

(d) Find all t-values for which $f'(t) = 0$.

interval $(0, 2)$, $x = 3.5$, $x = 5.2$, $x = 6.1$

(e) In the open interval $(0, 7)$, when does $g(x)$ have a maximum? A minimum?

$x = 3$ max, $x = 4$ min

(f) When is $g(x)$ increasing?

$[0, 3) \cup (4, 7)$

4. Find dy/dx for $y = \int_1^{\cos(x)} (1 + s^3)e^s \, ds$.

$$\frac{dy}{dx} = \left((1 + (\cos x)^3) \, e^{\cos x} \right) (-\sin x)$$

5. A bacteria population is 4000 at time $t = 0$ and its rate of growth is $1000 \times e^{t/2}$ bacteria per hour after t hours. What is the population after 4 hours?

$$P(t) = 4000 + \int_0^4 1000 e^{t/2}$$

$$= 4000 + \left[1000 \left(2e^{t/2} \right) \right]_0^4 = 4000 + 1000 \left(2e^2 - 2e^0 \right)$$

$$= 4000 + 1000 \left(2 (e^2 - 1) \right) \text{ bacteria.}$$

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$$= 4000 + \left[1000\left(2e^{t/2}\right)\right]_0^4 = 4000 + 1000\left(2e^2 - 2e^0\right)$$

$$= 4000 + 1000(2(e^2-1)) \text{ bacteria.}$$

6. What, if anything, is wrong with the following calculation?

$$\int_0^5 \frac{1}{x-2} dx = \ln(|x-2|)\Big|_0^5 = \ln(3) - \ln(2)$$

$f(x) = \frac{1}{x-2}$ has a vertical asymptote at $x=2$, right in the middle of the interval: $[0,5]$. Since f is not continuous on $[0,5]$, FTC part 2 doesn't apply.