FTC Part I:

$$\frac{d}{dx}\int_a^x f(s)\,ds = f(x) \qquad \text{(if } f(x) \text{ is continuous)}$$

FTC Part II:

If $F'(x) = f(x)$,

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Because of the FTC there is a strong connection between antiderivatives and definite integrals. This motivates the following notation:

$$\int f(x)\,dx = F(x)$$

means $F'(x) = f(x)$.

Or equivalently, $F(x)$ is an antiderivative of $f(x)$.

Of course, if you find one antiderivative, you can always add a constant. So it is traditional to write

$$\int f(x)\,dx = F(x) + C$$

Some texts think of $\int f(x)dx$ as meaning a whole family of antiderivatives. But we'll just use the original meaning:

$$\int f(x)\,dx = F(x) + C \qquad \text{means}$$

$$\frac{d}{dx}\Big(F(x) + C\Big) = f(x).$$

Do yourself a favor: add the $+C$ to remind yourself you have the freedom to add a constant.

The notation is historical and a little unfortunate:

$$\int f(x)\,dx \ : \quad \text{Indefinite integral}$$

Is a function, or a family of functions

$$\int_a^b f(x)\,dx \ : \quad \text{Definite integral}$$

Is a number

Connection: To compute

$$\int_a^b f(x)\,dx \, , \text{ if}$$

$$\int f(x)\,dx = F(x) \quad \text{then}$$

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Another perspective on FTC II

If $F'(x) = f(x)$

$$\int_a^b \underset{\uparrow \atop F'(x)}{f(x)}\,dx = F(b) - F(a)$$

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

"If you integrate a <u>rate</u> of change, you get a <u>net</u> change."

Your text calls this the Net Change Theorem.

E.g. If the rate of change of height of a ball is $h'(t)$, the net change in height from $t=1$ to $t=3$ is

$$\int_1^3 h'(t)\,dt = h(3) - h(1).$$

(See worksheet for concrete examples)