Where does exponential grantle come from ?

 $f(t) = C_o 2^{t/a}$  $(a \rightarrow f(a))$ a  $\rightarrow$  time to dentale: f(t+a) = 2 f(t) a lurays.Since  $2 = e^{\ln(2)}$  $2^{\frac{t}{a}} = (e^{\ln(2)})^{\frac{t}{a}} = e^{t \ln(2)/a}$ I.e.  $f(t) = C_0 e^{kt}$   $k = \underline{l_n(2)}$  in our case. Any exponential oranith / decay function has the form f (t) = Coekt for constants Co, E. Here's the deal:  $f'(t) = k[c_0 e^{kt}] = kf(t)$ rute of gruntly / decy proportional to amount of stuff

E.g., in a petri dish of buckering

the granth mate (buckers per minute) is proportional to the number of bactern (at least ontil competition for food sets in).

Subtle variation: Newtons hav of Cooling.

Object: cossee, house, corpse has top T(E) Environment has tomp To.

Newton's low of coolary states

 $T'(t) = k(T - T_0)$ 

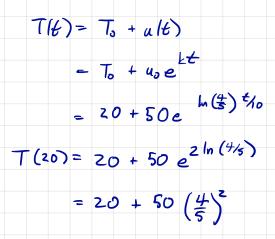
rate of change of temp is proportional to

he tempenture difference between the object

and its surrendings.

1) T'= O. f T= To (no churse if some temp) 2) T'40 A T>T. (T-7.70 .0 k(T-7.)<0 implies k < 0. 3) u(t) = T(t) - To (tomp with a new 0)  $u'(t) = T'(t) = k(T - T_0) = ku.$ u(t) = uo ekt uo some constant. 4) Since k < 0,  $\lim_{k \to \infty} e^{k\xi} = 0$  and  $\lim_{k \to \infty} u(\xi) = 0$ . Since  $T(t) = u(t) + T_0$ ,  $\lim_{t \to \infty} T(t) = 0 + T_0 = T_0$ (eventul temp is surroudings) s)  $u_0 = T(0) - T_0 = 70^\circ - 20^\circ = 50^\circ C$ 6)  $u(t) = u_0 e^{kt} = 50e^{kt}$ T(10) = 60 - u(10) = 40.40=50 e10k ln(==10k  $k = \frac{1}{10} \ln \left(\frac{4}{5}\right) \quad k = -0.0723$ 

7)  $u(t) = u_0 e^{kt}$ 



= 52°C