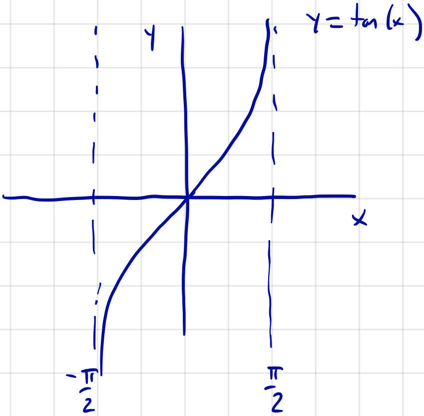
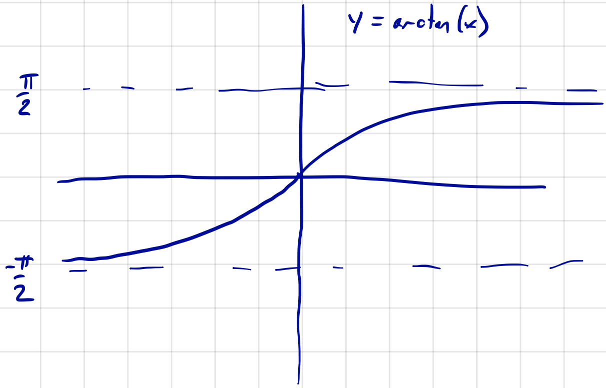
Inverse Trig functions:





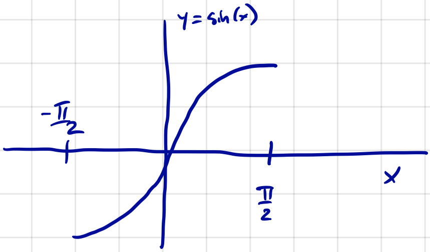
Rules: $\tan(\arctan z) = z$ always

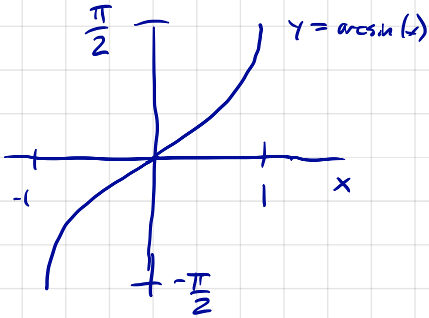
$\arctan(\tan\theta) = \theta$ so long as $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

(otherwise $\arctan(\tan\theta) = \theta - k\pi$ for $\theta - k\pi \in (-\frac{\pi}{2}, \frac{\pi}{2})$).

There is <u>always</u> an angle restriction for inverse trig functions.

Text uses $\tan^{-1} z$. Ugh.

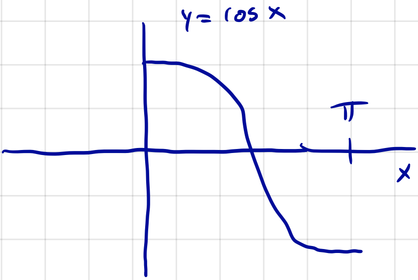


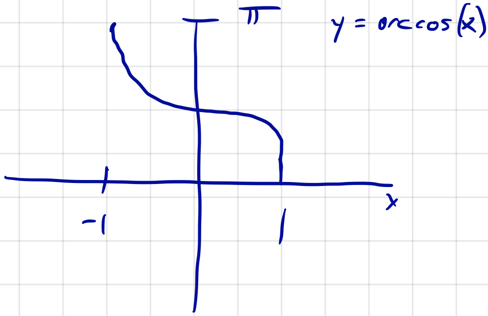


$\sin(\arcsin(z)) = z \qquad -1 \leq z \leq 1$

$\arcsin(\sin\theta)) = \theta \qquad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

↳ otherwise is $\theta - k\pi$





$\cos(\arccos(z)) = z$ always, $-1 \leq z \leq 1$

$\arccos(\cos(\theta)) = \theta \qquad 0 \leq \theta \leq \pi$

$\theta - k\pi$ otherwise

to find derivatives of these inverse functions:

$$y = \arcsin(x)$$

$$\sin(y) = x$$

$$\cos(y)\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos(y)}$$

$$\cos^2(y) + \sin^2(y) = 1$$

$$\cos^2(y) = 1 - x^2$$

$$\cos(y) = \pm\sqrt{1-x^2}$$

$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ so $\cos(y) \geq 0$

$$\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$$

$$y = \arctan(x)$$

$$\tan y = x$$

$$\sec^2 y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$\sec^2(y) - \tan^2(y) = 1$$

$$\frac{1}{\cos^2(y)} - \frac{\sin^2(y)}{\cos^2(y)} = \frac{\cos^2(y)}{\cos^2(y)} = 1 \checkmark$$

$$\sec^2(y) = 1 + \tan^2(y) = 1 + x^2$$

$$\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$$

Via a similar exercise:

$$\frac{d}{dx} \arccos(x) = \frac{-1}{\sqrt{1-x^2}}$$