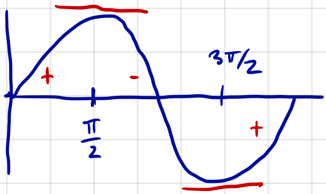
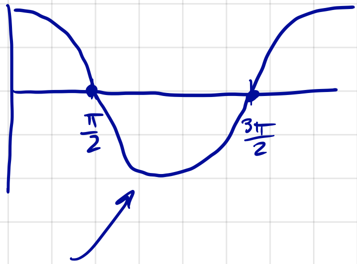
Two more derivatives:





looks kinda like cos(x).

But kinda isn't exact.

let's show $\frac{d}{dx}\sin(x) = \cos(x)$ exactly.

Keys: 1) $\lim\limits_{x \to 0} \frac{\sin(x)}{x} = 1$ $\qquad \frac{0}{0}$

2) $\lim\limits_{x \to 0} \frac{1-\cos(x)}{x} = 0$ $\qquad \frac{0}{0}$

Assuming these for the moment, we'll also need

3) $\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cos(A)$ ← trig identity.

4) $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

With this:

$$\frac{d}{dx}\sin(x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

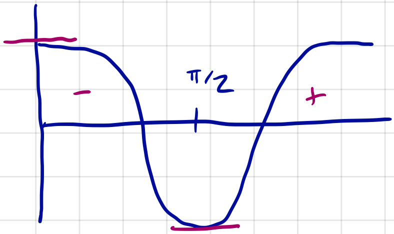
$$= \lim_{h\to 0} \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \left[\sin(x)\frac{[\cos(h) - 1]}{h} + \cos(x)\frac{\sin(h)}{h} \right]$$

$$= \sin(x) \lim_{h\to 0} \frac{[\cos(h)-1]}{h} + \cos(x) \lim_{h\to 0} \frac{\sin(h)}{h}$$

$$= \sin(x)\cdot 0 + \cos(x)\cdot 1$$

$$= \cos(x).$$



$$\lim_{h \to 0} \frac{\cos(x+h) - \cos(x)}{h} = \lim_{h \to 0} \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h}$$

$$= \lim_{h \to 0} \cos(x) \frac{[\cos(h) - 1]}{h} - \lim_{h \to 0} \sin(x) \frac{\sin(h)}{h}$$

$$= \cos(x) \cdot 0 - \sin(x) \cdot 1$$

$$= -\sin(x)$$

So it boils down to two limits:

a) $\lim_{\cancel{x} \to 0} \frac{\sin(x)}{x} = 1$

b) $\lim_{x \to 0} \frac{\cos(x) - 1}{x} = 0.$

In fact, if I can get a) I'll also get b).:

$$\sin^2(x) + \cos^2(x) = 1$$

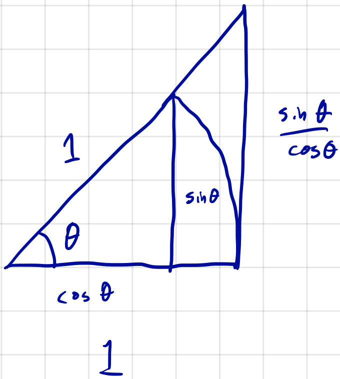
$$\sin^2(x) = 1 - \cos^2(x)$$
$$= (1 - \cos(x))(1 + \cos(x))$$

$$1 - \cos(x) = \frac{\sin^2(x)}{1 + \cos(x)}$$

$$\frac{1 - \cos(x)}{x} = \frac{\sin(x)}{x} \quad \frac{\sin(x)}{1 + \cos(x)} \qquad (x \neq 0)$$

$$\lim_{x \to 0} \frac{1 - \cos(x)}{x} = 1 \cdot \frac{\sin(0)}{1 + 1} = 0$$

So now it boils down to $\lim_{x \to 0} \frac{\sin(x)}{x} = 0.$



area: $\frac{\theta}{2\pi} \cdot \pi = \frac{\theta}{2}$

$$\frac{1}{2} \cos\theta \sin\theta \leq \frac{\theta}{2} \leq \frac{1}{2} \frac{\sin\theta}{\cos\theta}$$

$$\cos\theta \leq \frac{\sin\theta}{\theta} \leq \frac{1}{\cos\theta}$$

As $\theta \to 0$, $\cos\theta \to 1$, $\frac{1}{\cos\theta} \to 1$, so $\frac{\sin\theta}{\theta} \to 1$.

Other trig derivatives:

$$\frac{d}{dx}\tan(x) = \frac{d}{dx}\frac{\sin(x)}{\cos(x)} = \sec^2(x)$$ (You did this!)

$$\frac{d}{dx}\cot(x) = \frac{d}{dx}\frac{\cos(x)}{\sin(x)} = \frac{-\sin^2 x - \cos^2(x)}{\sin^2(x)} = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \frac{d}{dx}\frac{1}{\cos(x)} = -\frac{-\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x)}\,\frac{1}{\cos(x)} = \sec x \tan x$$

$$\frac{d}{dx}\sec(x) = \frac{d}{dx}\frac{1}{\sin(x)} = -\frac{\cos(x)}{\sin^2(x)} = -\cot x \csc(x)$$

Worksheet?