Last Class

Product rule $\frac{d}{dx}\left[f(x)g(x)\right] = \left(\frac{d}{dx}f(x)\right)g(x) + f(x)\frac{d}{dx}g(x)$

$$f(x+h)g(x+h) - f(x)g(x) = f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - \ldots$$

$$= \left(f(x+h) - f(x)\right)g(x+h) + f(x)\left(g(x+h) - g(x)\right)$$

$$\lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} g(x+h)$$

$$+ \lim_{h\to 0} f(x) \frac{g(x+h) - g(x)}{h}$$

$$= f'(x)g(x) + f(x)g'(x).$$

Justify!

Application: $\frac{d}{dx}x^1 = 1 \cdot x^0$

$\frac{d}{dx}x^2 = 2x^1$

$\frac{d}{dx}x^3 = 3x^2$

$\frac{d}{dx}x^n = (n-1)x^n$

$\frac{d}{dx}x^{n+1} = \frac{d}{dx} x x^n$

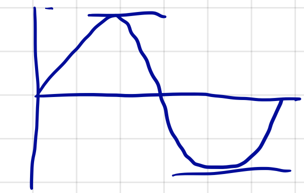
$= x^n + x n x^{n-1}$

$= (n+1)x^n$

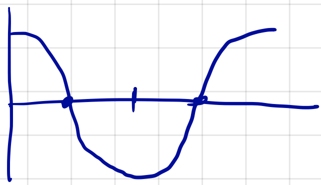
Inverse rule: $\frac{d}{dx} \frac{1}{f(x)} = - \frac{f'(x)}{f(x)^2}$

Quotient rule $\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{f(x)^2}$

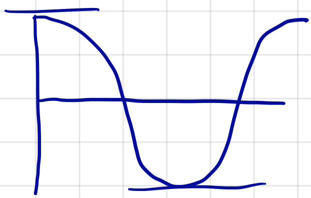
Two more derivatives:



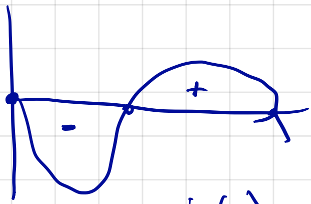
$\sin(x)$



$\cos(x)$



$\cos(x)$



$-\sin(x)$

In fact: $\frac{d}{dx} \sin(x) = \cos(x)$

$\frac{d}{dx} -\cos(x) = \sin(x)$.

$\boxed{\text{Will justify next.}}$