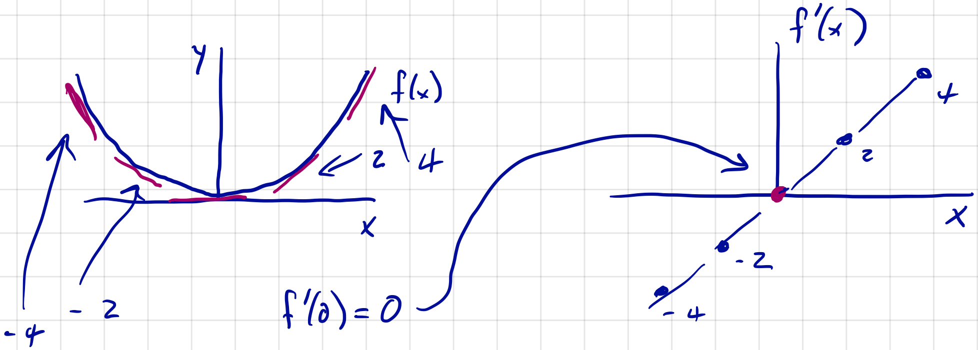
A perspective on the derivative

At each x, $f'(x)$ is the slope of the tangent line at x.

↳ new function; it depends on x, too!



How does this work? $f(x) = x^2$.

$$f'(1) = \lim_{b \to 1} \frac{b^2 - 1^2}{b-1}$$

$$= \lim_{b \to 1} \frac{(b-1)(b+1)}{b-1}$$

$$= \lim_{b \to 1} b+1 = 2$$

$$f'(2) = \lim_{b \to 2} \frac{b^2 - 4}{b - 2}$$

$$= \lim_{b \to 2} \frac{(b-2)(b+2)}{b-2}$$

$$= \lim_{b \to 2} b + 2 = 4$$

$$f'(a) = \lim_{b \to a} \frac{b^2 - a^2}{b - a}$$

$$= \lim_{b \to a} \frac{(b-a)(b+a)}{b-a}$$

$$= \lim_{b \to a} b + a$$

$$= a + a = 2a$$

Or use x instead of a:

$$f'(x) = \lim_{b \to x} \frac{b^2 - x^2}{b - x}$$

$$= \lim_{b \to x} \frac{(b-x)(b+x)}{b-x}$$

$$= \lim_{b \to x} b + x = x + x = 2x.$$

$$\boxed{f'(x) = 2x}$$