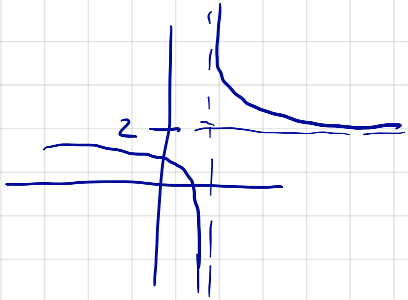
Limits at ∞.

$$\frac{1}{x-1} + 2 = \frac{2x-1}{x-1}$$



When x is large, $\frac{2x-1}{x-1} \approx \frac{2x}{x} = 2$.

(2 · 1billion − 1 ≈ 2 · 1 billion!)

We'll express this via $\lim\limits_{x\to\infty} \frac{2x-1}{x-1} = 2$.

Here's how we can justify:

top $\to \infty$
bottom $\to \infty$

$\frac{\infty}{\infty} \to$ indeterminate

$\frac{0}{0} \to$ also

Instead:

$$\lim_{x\to\infty} \frac{2x-1}{x-1} = \lim_{x\to\infty} \frac{2-\frac{1}{x}}{1-1/x}$$

(multiply top + bottom by $\frac{1}{x}$)

Now: $\lim_{x \to \infty} \frac{1}{x} = 0$.

So $$\lim_{x\to\infty} \frac{2-\frac{1}{x}}{1-\frac{1}{x}} = \frac{\lim_{x\to\infty} 2-\frac{1}{x}}{\lim_{x\to\infty} 1-\frac{1}{x}} = \frac{2}{1}$$

Facts you will need:

① $$\lim_{x\to\infty} \frac{1}{x} = 0$$

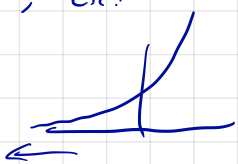
$$\lim_{x\to\infty} \frac{1}{x^2} = 0 \qquad \lim_{x\to\infty} \frac{1}{x^n} = 0 \quad n>0.$$

$$\lim_{x\to\infty} \frac{1}{5x} = 0$$

Also, $\lim_{x\to -\infty} \frac{1}{x} = 0$

$\frac{1}{x^2} = 0$, etc.

② $$\lim_{x\to -\infty} 10^x = 0$$



3:

$$\lim_{x \to \infty} \arctan(x) = \frac{\pi}{2}$$

$$\lim_{x \to -\infty} \arctan(x) = -\frac{\pi}{2}$$

{Read text about relation between asymptotes + infinite limits}.