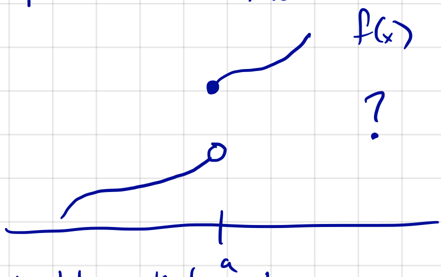
Continuity.

Does temperature ever do this:

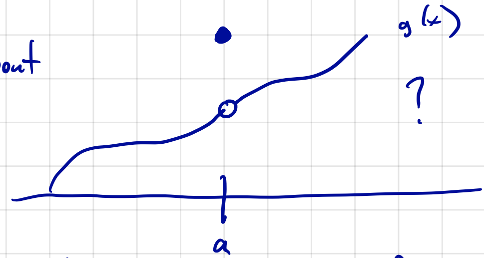
a)



We don't like that gap!

b)

How about



Nah! These are examples of <u>discontinuous</u> functions.

a) $\lim_{x \to a} f(x)$ does not exist

b) $\lim_{x \to a} g(x)$ exists, but does not equal $g(a)$.

Def: A function $f(x)$ is continuous if at each a in its domain,

$$\lim_{x \to a} f(x) = f(a).$$

I.e. we can compute limits by direct substitution.

E.g. polynomials, rational functions, root functions — from sec 2.2

trig functions, exponential functions, log functions — my promise to you

What about $\lim_{x \to 3} (x^2-2) + 10^x = \lim_{x\to 3}(x^2-2) + \lim_{x \to 3} 10^x$

$= (3^2-2) + 10^3$

Indeed, a sum of continuous functions is cts.

Similarly for a difference or product. For division, a little care:

$$\lim_{x \to 3} \frac{10^x}{x^2-2} = \frac{\lim_{x\to 3} 10^x}{\lim_{x \to 3} x^2-2} = \frac{10^3}{3^2-2}$$

← ok since this isn't 0.

There are four variations of continuity you should be aware of

1) continuity at a point (Def 1 in text)

2) one-sided continuity (Def 2 in text)
(left-right)

Rules for composition

$$\lim_{x\to 3} \sin\left(\sqrt{x^2+1}\right) = \sin\left(\sqrt{3^2+1}\right)$$

Direct subs.

Why?

$$\lim_{x\to a} f(g(x)) = f(g(a)) \text{ if } f, g \text{ are continuous.}$$

"A composition of continuous functions is continuous!"

$$\lim_{x\to 3} \sqrt{x^2+1} = \sqrt{3^2+1} = \sqrt{10}$$

$\sqrt{x}$ $\quad x^2+1$

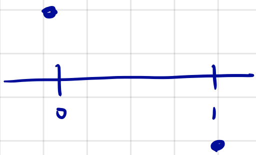
$$\lim_{x\to 3} \sin\left(\sqrt{x^2+1}\right) = \sin\left(\sqrt{10}\right)$$

Important theorem:

Consider $x^5 - 3x + 1 = p(x)$

$p(0) = 1$

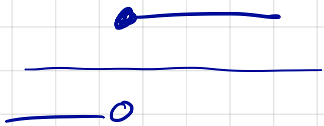
$p(1) = -1$



Somewhere in $[0,1]$ is a spot x where $p(x) = 0$.

This doesnt work for discontinuous functions

e.g. $f(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$



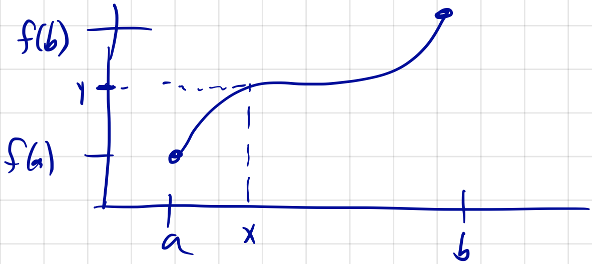
$f(x) \neq 0$ ever!

Intermediate Value Theorem

If $f(x)$ is a continuous function defined on an interval $[a,b]$, for any y between $f(a)$ and $f(b)$

there is $x \in [a,b]$ with $f(x)=y$.

[In particular, if $f(a) \geq 0$ and $f(b) \leq 0$ there is x in $[a,b]$ with $f(x)=0$.]



(!)

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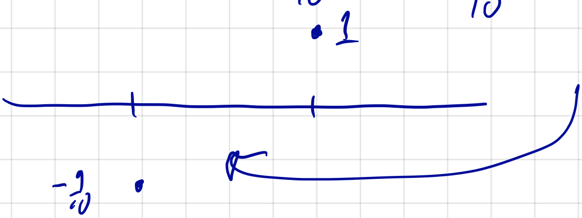
e.g. is there a number x with $10^x = x^2$

$f(x) = 10^x - x^2$ Want $f(x)=0$.

$f(0) = 1$

$f(-1) = \frac{1}{10} - 1 = -\frac{9}{10}$

Aha!



there's a root somewhere in here!