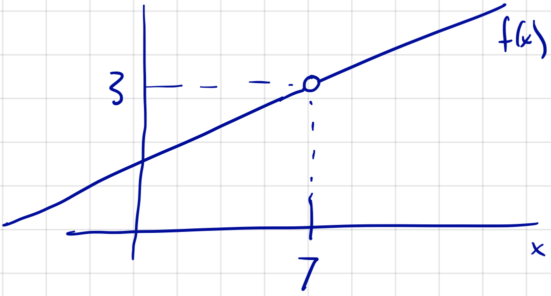
Last class:

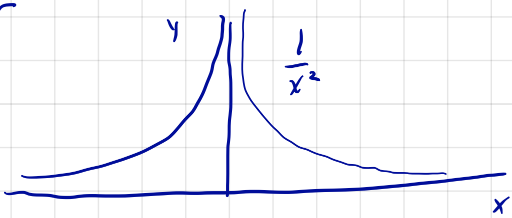
Picture:



$$\lim_{x \to 7} f(x) = 3$$

There are some variations on the limit theme you need to know about:

Consider

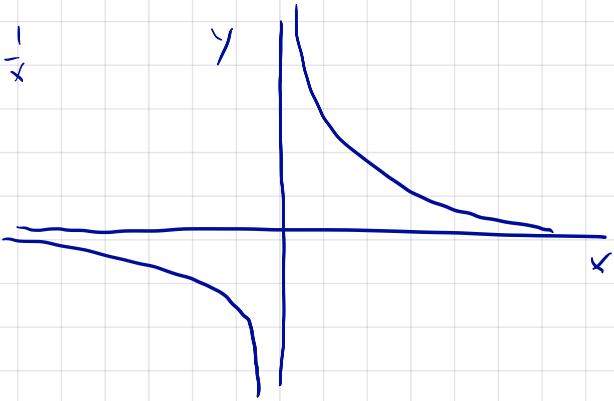


We'll say $\lim_{x \to 0} \frac{1}{x^2} = \infty.$

$-\infty$ is also possible

($\lim_{x \to a} f(x) = \infty$ if the values of f can be made as large (and positive) as you wish taking x close to a (but $x = a$ is not required))

$f(x) = \frac{1}{x}$



$\lim_{x \to 0} \frac{1}{x}$ does not exist.

But we have one-sided limits:

$\lim_{x \to 0^+} \frac{1}{x} = \infty$ ← from the right

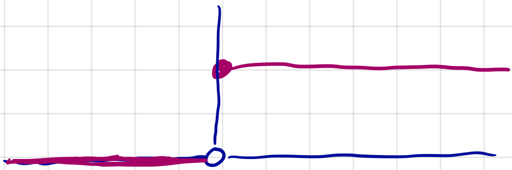
(only $x > 0$ is under consideration)

$\lim_{x \to 0^-} \frac{1}{x} = -\infty$ ← from the left

We can also have one-sided limits in other contexts:

Heaviside function

$$H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$



$$\lim_{x \to 0^+} H(x) = 1$$

$$\lim_{x \to 0^-} H(x) = 0$$

Important fact: if $\lim_{x \to a^+} f(x) = L$ and $\lim_{x \to a^-} f(x) = M$

1) If $L = M$ $\lim_{x \to a} f(x) = L \ (= M)$

2) If $L \neq M$, $\lim_{x \to a} f(x)$ d.n.e.