Last class:

Population of caribou:

$$p(t) = 1000(1.1)^t$$

Note: this population grows by 10% over any one year period:

$$p(t+1) = 1000\,(1.1)^{t+1}$$

$$= 1000\,(1.1)^t\,(1.1)$$

$$= p(t)\,(1.1)$$

$$= p(t)\left(1 + \frac{1}{10}\right)$$

$$= p(t) + \frac{1}{10}\,p(t)$$

(t is anything, not just $0, 1, 2, \ldots$)

Q: When will the population be 2000 (double the starting population)?

$$1000\,(1.1)^t = 2000$$
$$(1.1)^t = 2$$

Now what? We need a function that undoes exponentiation, and these are logarithms.

Recall

$$\log_{10}(10^x) = x$$
$$10^{\log_{10}(y)} = y$$

10^x and $\log_{10} x$ undo each other, and are called inverse functions.

Other bases are possible: $\log_2 2^x = x$

$\log_e e^x = x$

$\hookrightarrow \log_e = \ln$

exponent rules have companion log rules.

$$10^{x+y} = 10^x 10^y \qquad \log_{10}(ab) = \log_{10}(a) + \log_b(b)$$

$$(10^x)^y = 10^{xy} \qquad \log_{10} a^b = b \log_{10}(a)$$

$$10^0 = 1 \qquad \log_{10}(1) = 0$$

For many purposes, you are free to pick any base (your calcs will have $\log_{10}$ and $\ln = \log_e$, but not others)

In fact $\log_2$ and $\log_{10}$ are related:

$$\log_{10}(2^x) = x \log_{10} 2$$

$$\log_2(2^x) = x$$

substitute $2^x = y$

$x = \log_2(y)$

$$\log_{10}(y) = \log_2(y) \log_{10}(2)$$

$$\boxed{\log_2(y) = \frac{\log_{10}(y)}{\log_{10}(2)}}$$

This works for other bases, too.

Back to the problem:

$$(1.1)^t = 2$$

$$\ln\left((1.1)^t\right) = \ln(2)$$

$$t \ln(1.1) = \ln(2)$$

$$t = \underbrace{\ln(2)/\ln(1.1)}_{b} \approx 7.27.$$

$$(1.1)^b = 2$$

$$1.1 = 2^{1/b}$$

$$1.1^t = 2^{t/b}$$

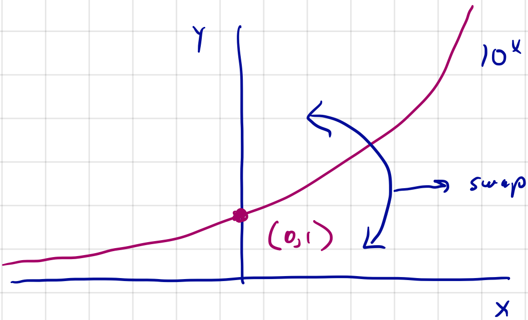
$$p(t) = 1000(1.1)^t = 1000\left(2^{t/b}\right).$$

$t = b$, $p = 2000$

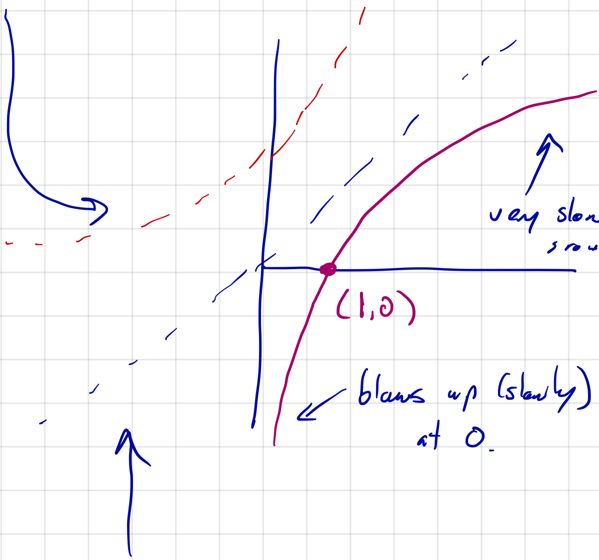
$t = 2b$ $p = 4000.$ etc.

$\log_{10}(x)$ is the inverse function of 10^x

how to graph it:



swap the roles of x + y for an inverse function.



very slow growth

blows up (slowly) at 0.

not defined here.