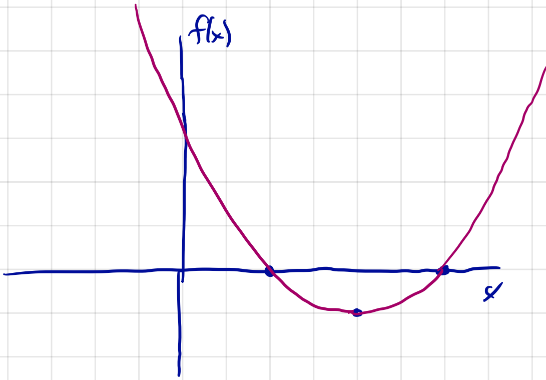
Scaling Transformations:

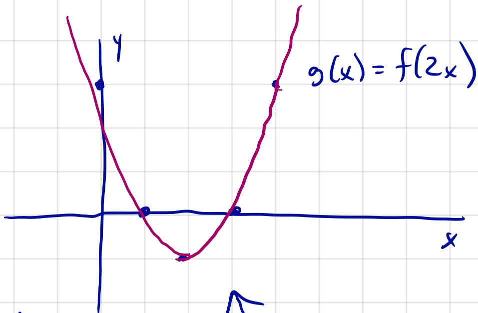
$f(x) = \frac{1}{4}(x-2)(x-6)$



landmarks:
$f(0) = 3$
$f(2) = 0$
$f(6) = 0$
$f(4) = -1$

Scaled variation: $g(x) = f(2x)$

landmarks:
$g(0) = f(0) = 3$
$g(1) = f(2\cdot 1) = 0$
$g(3) = f(6) = 0$
$g(2) = f(4) = -1$

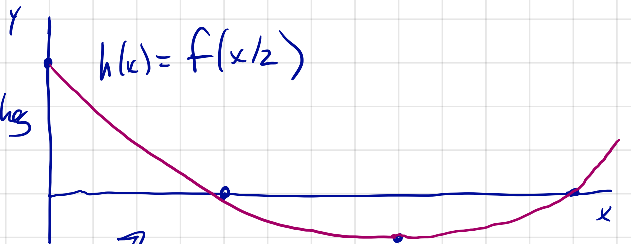


kinda counter intuitive: 2 squeezes

variation: $h(x) = f(x/2)$

$\hookrightarrow \frac{1}{2}$ stretches by 2.

$h(0) = f(0) = 3$
$h(4) = f(2) = 0$
$h(12) = f(6) = 0$
$h(8) = f(4) = -1$



Exponential functions.

Examples	Rules:
$5^6 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ $= (5 \cdot 5 \cdot 5 \cdot 5) \cdot (5 \cdot 5)$ $= 5^4 \cdot 5^2$	$(r > 0,\ a, b \in \mathbb{R})$ $r^a r^b = r^{a+b}$
$(5^2)^3 = 5^2 \cdot 5^2 \cdot 5^2$ $= 5^{2+2+2}$ $= 5^6$	$(r > 0,\ a, b \in \mathbb{R})$ $(r^a)^b = r^{ab}$
$(2 \cdot 7)^3 = (2 \cdot 7)(2 \cdot 7)(2 \cdot 7)$ $= (2 \cdot 2 \cdot 2)(7 \cdot 7 \cdot 7)$ $= 2^3 \cdot 7^3$	$(r > 0,\ s > 0,\ a \in \mathbb{R})$ $(rs)^a = r^a s^a$

Consequences of the rules:

$$r^0 = 1 \qquad (r > 0)$$

$$r^{-1} = \frac{1}{r} \qquad (r > 0)$$

$$1 = r^0 = r^{1+(-1)} = r^1 r^{-1} = r r^{-1}$$

$$r r^{-1} = 1 \Rightarrow r^{-1} = 1/r.$$

Two kinds of related functions:

a) $f(x) = x^3$ (power functions. $\sqrt{x}$, $x^{2/3}$, x^4, etc.)

b) $f(x) = 3^x \longrightarrow$ exponential functions.

Exponential functions occur in many physical applications involving doubling (or halving) in a fixed time period.

e.g. A population of caribou grows by 10% per year, and has 1000 animals at time $t = 0$ years.

Claim: $p(t) = 1000\,(1.1)^t$

Did this work?

$p(0) = 1000 \cdot (1.1)^0 = 1000$ ✓

$p(1) = 1000\,(1.1) = 1000\left(1 + \frac{1}{10}\right)$

$= 1000 + \underbrace{\frac{1}{10}\,1000}_{10\%!}$

$= 1100.$

$p(2) = 1000\,(1.1)^2$

$= 1000\,(1.1)\cdot(1.1)$

$= p(1)\,(1.1)$

$= (1100)\left(1 + \frac{1}{10}\right) = 1100 + \overbrace{\frac{1}{10}\,1100}^{10\%!}$

$= 1210$

How many caribou after 1 year, 6 months?

$$p(1.5) = 1000\,(1.1)^{1.5} = 1153.6897\ldots$$

don't take this too seriously!

Where is the doubling?

Consider:

$f(x) = 2^x$: $f(1) = 2$ doubles
$f(2) = 4$ doubles
$f(3) = 8$

How about $f(x) = 2^{x/3}$?

$f(0) = 1$ doubles
$f(3) = 2^{3/3} = 2$ doubles
$f(6) = 2^{6/3} = 2^2 = 4$ doubles
$f(9) = 8$

When x goes up by 3 this function doubles.

Moreover: $2^{x/3} = (2^{1/3})^x$

$\approx (1.26)^x$

The function $f(x) = (1.26)^x$ doubles when x goes up by about 3.

round off error

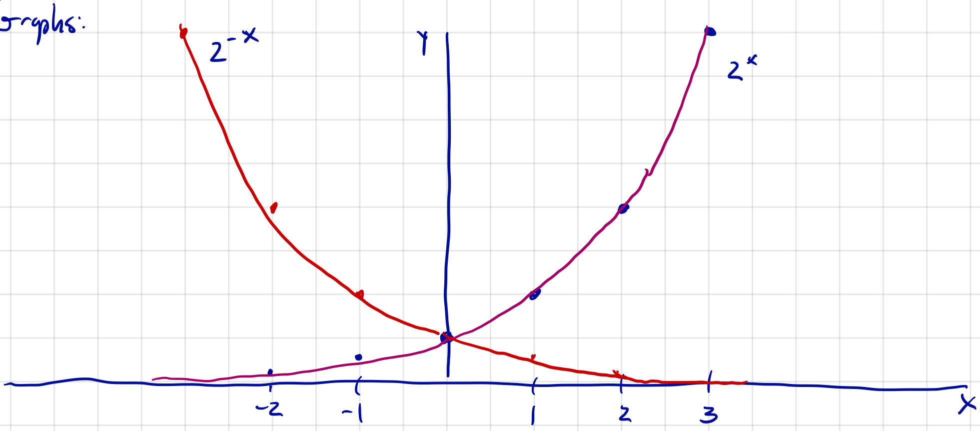
Doubling includes halving:

e.g. $f(x) = 2^{-x}$: $f(1) = \frac{1}{2}$

$f(2) = \frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2}$, etc.

each time x goes up by 1, f is cut in $1/2$.

Graphs:



E.g. Plutonium 241 has a half life of 14.4 years. If we start with 10 g of Pl_{241}, how much is left after 3 years?

Consider $$m(t) = C\,2^{-t/h}$$

Then $$m(0) = C\,2^{0} = C$$

$$m(h) = C \cdot 2^{-h/h} = C \cdot 2^{-1} = C/2$$

$$m(2h) = C\,2^{-2h/h} = C \cdot 2^{-2} = C/4$$

This function starts at C at $t=0$ and has values cut in half every time t goes up by h.

So if you know the initial quantity and the half life, you can use it!

$$m(t) = 10\,2^{-t/14.4}$$

So $$m(3) = 10\,2^{-3/14.4} \approx 8.65\text{g}$$