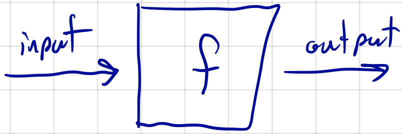
The modern notion of a function is a recent innovation in mathematics (early 20th century).

For our purposes → Think of as a box



Rules: a) if you input the same thing twice, the same output comes out

b) Only one output comes out.

E.g. stock price at the end of trading of FB = facebook.

ask
- Inputs: day
- Output: price (in dollars)

Functions have a notion of

- domain → allowable inputs
  super important

- range → allowable outputs,
  has flexibility

For the example, domain is trading days since May 18, 2012 (IPO).

range: → $\mathbb{R}$ all real numbers
$\mathbb{Q}$ all rational numbers
$\mathbb{Q}_+$ all positive rational numbers
(and we can shrink)

Your book usually takes range to mean "all possible outputs and <u>only</u> those".

I'll clarify shortly.

This function (FB) is not typical for us because it is discrete: no partial days!

In calculus, we'll deal with functions of a 'continuous variable'.

e.g. Temperature at FAI as a function of time
→ nice and continuous!

But, truth be told, we will model these 'real world' functions with mathematical idealizations.

e.g. (input →) $f(x) = \frac{1}{x}$ ← output

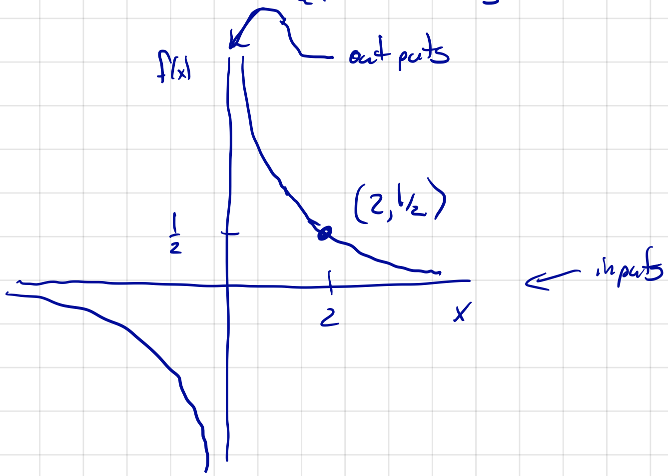
domain: informally: $x \neq 0$

formally: $(-\infty, 0) \cup (0, \infty)$

↓

$\{x : 0 < x < \infty\}$.

Graph:



range: $\mathbb{R}$

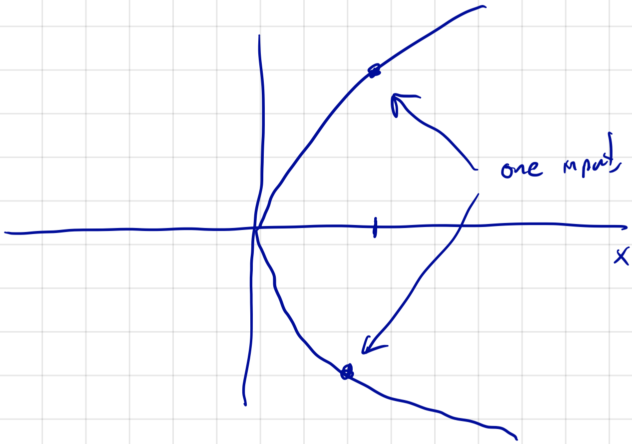
$\mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$ ← preferable, alas e.g.

$f(x) = \sqrt{x}$ note: $\sqrt{x}$ is a number y with $y^2 = x$.

e.g. $\sqrt{4}$ $y = 2$? $2^2 = 4$

$y = -2$ $(-2)^2 = 4$

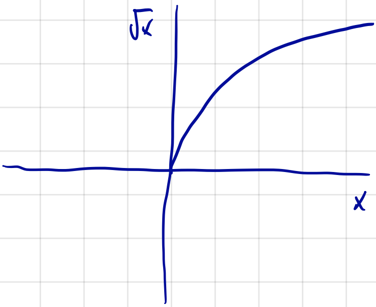
We can't have both:



one input and two outputs.

Our outputs will be **one** real number.

For us, $\sqrt{x}$ is always ≥ 0.



domain: $[0, \infty)$

range: $[0, \infty)$

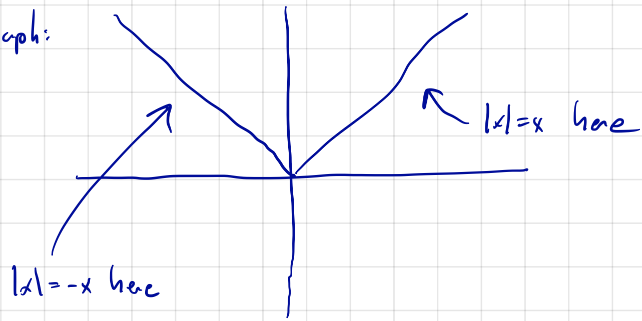
e.g: $f(x) = |x| = \begin{cases} x & : x \geq 0 \\ -x & : x < 0 \end{cases}$

↑ piecewise-defined function

$|-7| = 7$

$|\pi| = \pi$

Graph:



domain: $\mathbb{R}$

range: $[0, \infty)$

e.g. $f(x) = 10^x$

$$f(1) = 10$$

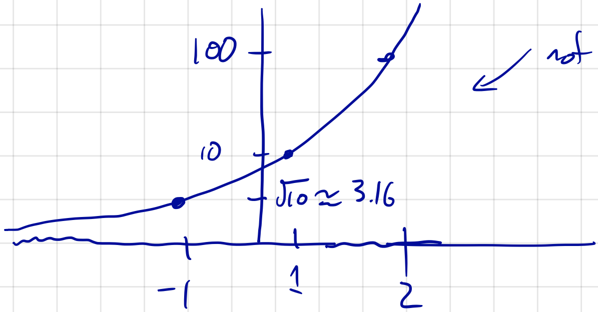
$$f(2) = 100$$

$$f\left(\tfrac{1}{2}\right) = 10^{1/2} = \sqrt{10}$$

$$f\left(1/3\right) = 10^{1/3} = \sqrt[3]{10} \quad \longrightarrow \quad \left(\sqrt[3]{10}\right)^3 = 10$$

$$f(3.14) = 10^{\frac{314}{100}} = \left(10^{\frac{1}{100}}\right)^{314}$$

$f(\pi) = ??$ you'd like to believe it's approximated by $10^{3.14159}$ and it is. you're probably not too worried about it!



not to scale!

domain: $\mathbb{R}$

range : $(0, \infty)$

Do worksheet 1.1