Name:

1. The temperature on metal plate is given by

$$T(x,y) = \frac{50}{1+x^2+y^2}$$

where T is measured in °C and x and y are measured in centimeters from the center of the plate.

1. Compute $\vec{\nabla} T(x,y)$.

$$\frac{\partial T}{\partial x} = \frac{-100x}{(1+x^2+y^2)^2} \qquad \nabla T = \frac{-100}{(1+x^2+y^2)^2}\langle x, y\rangle$$

$$\frac{\partial T}{\partial y} = \frac{-100y}{(1+x^2+y^2)^2}$$

2. At the point $P = (2,1)$ determine the direction $\mathbf{u}$ of maximum increase of the temperature. Express your answer as a unit vector.

∇T is parallel to $\langle -2, -1\rangle$ at P.

unit vector: $\vec{u} = \left\langle \frac{-2}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \right\rangle$

3. A bug is at $P = (2,1)$ and crawling with velocity $\mathbf{v} = \langle 0, 1\rangle$ centimeter/second. What is the rate of change in temperature that the bug sees? Your answer should have units of °C per second.

$$\nabla T \cdot \langle 0, 1\rangle = \frac{-100 \cdot y}{(1+4+1)^2} = \frac{-100}{36} \ {}^\circ C/s$$

2. Consider the surface given by $z = f(x, y)$ with

$$f(x, y) = x^2 - 2y^2.$$

At the point $P = (2, 3)$ we have:

$$\begin{aligned} f(2,3) &= -14 \\ f_x(2,3) &= 4 \\ f_y(2,3) &= -12. \end{aligned}$$

Determine the equation of the tangent plane to the surface at P.

$$z = -14 + 4(x-2) - 12(y-3)$$