Name:

1. Consider the function

$$f(x, y) = \sin(xy)$$

- Compute f_x.

$$f_x = \cos(xy)\, y$$

- Compute f_{xy}.

$$f_{xy} = \partial_y f_x$$

$$= \partial_y \left(\cos(xy)\, y \right)$$

$$= -\sin(xy)\, xy + \cos(xy)$$

- Compute f_{yx}.

$$f_{yx} = f_{xy} = -\sin(xy)\, xy + \cos(xy)$$

2. Suppose $z = f(x, y) = \ln(x - y^2)$. Use the total differential $dz = f_x\,dx + f_y\,dy$ to estimate $f(5.1, 2.2)$. You'll perhaps find it helpful to observe that $f(5, 2) = 0$.

$$f_x = \frac{1}{x-y^2} \qquad f_y = \frac{-2y}{x-y^2}$$

$$f_x(5,2) = \frac{1}{5-4} = 1 \qquad f_y(5,2) = \frac{-4}{5-4} = -4$$

$$f(5.1, 2.2) = f(5,2) + f_x \cdot 0.1 + f_y \cdot 0.2$$

$$= 0 + 1 \cdot 0.1 - 4 \cdot 0.2 = -0.7$$

3. Suppose you know

$$\frac{\partial z}{\partial x} = 3, \quad \frac{\partial z}{\partial y} = -2, \quad \frac{dx}{dt} = -3, \quad \frac{dy}{dt} = 4.$$

Compute dz/dt.

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$= 3 \cdot (-3) + (-2) \cdot 4$$

$$= -9 + -8 = -17$$