Name: **ID:**   .
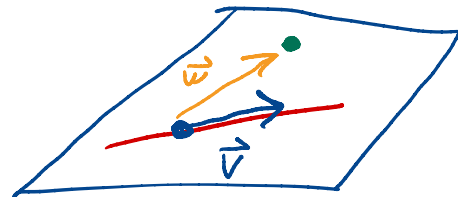
1.

    a) Determine the equation of a plane that passes through the point $(3, -2, 3)$ and the line $\mathbf{r}(t) = \langle 1, 2, 2 \rangle + t\langle -1, 4, 2 \rangle$.

$$\vec{v}$$

$$\vec{W} = \langle 3, -2, 3 \rangle - \langle 1, 2, 2 \rangle$$

$$= \langle 2, -4, 1 \rangle$$

$$\vec{n} = \vec{v} \times \vec{w} \longrightarrow \begin{array}{ccc} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 4 & 2 \\ 2 & -4 & 1 \end{array} \longrightarrow$$

$$\vec{n} = (4+8)\hat{\imath} - (-1-4)\hat{\jmath} + (4-8)\hat{k}$$

$$= \langle 12, 5, -4 \rangle$$

$$\boxed{12(x-1) + 5(y-2) - 4(z-2) = 0}$$

    b) Determine the equation of the plane parallel to the one you just found, but passing through the origin.

$$\boxed{12x + 5y - 4z = 0}$$

2. Determine a function $\mathbf{r}(t)$ that traces a circle of radius 3 centered at the point $(0, 3)$. The circle should be traced out clockwise and should go around once for $0 \le t \le \pi$. Note the upper end for t in this range!

$$\vec{r}(t) = \langle 3\cos(-2t), 3 + 3\sin(-2t) \rangle$$

$3\cos(-2t)$ — from center at $\langle 0, 3 \rangle$

$3\cos(-2t)$

↑ radius

↑ — makes time run backwards

2 makes time run twice as fast

3.

  a) A bug wanders in the plane with a path $\mathbf{r}(t) = \langle t^3, t \rangle$ cm with time $-1 \le t \le 1$ seconds. Sketch the path of the bug below, being careful to show correct behavior at time $t = 0$.

y, cm

-1

x, cm

displacement

$x = y^3$

(like $y = x^3$ but swap axes)
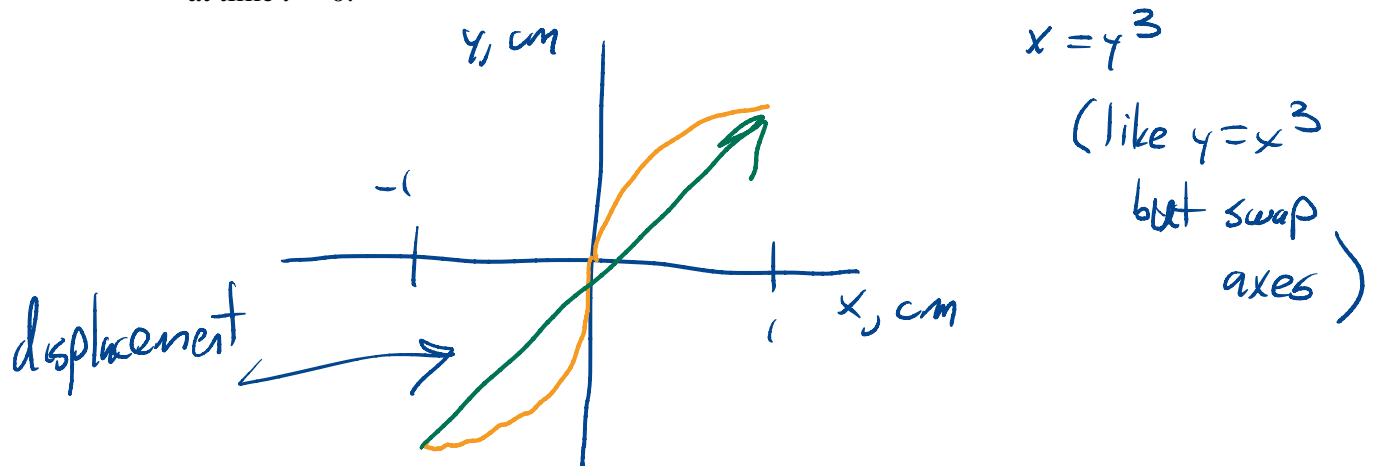
  b) Compute the displacement vector from $t = -1$ to $t = 1$ and add it to your diagram. Units please.

$$\vec{r}(1) = \langle 1, 1 \rangle$$

$$\vec{r}(-1) = \langle -1, -1 \rangle$$

$$\vec{r}(1) - \vec{r}(-1) = \langle 2, 2 \rangle \text{ cm}$$

  c) Compute the average rate of change of position with respect to time from $t = -1$ to $t = 1$ seconds. Include units in your answer.

$$\frac{1}{1 - (-1)} \langle 2, 2 \rangle = \boxed{\langle 1, 1 \rangle \text{ cm/s}}$$