Name: Solutions

Stokes's Theorem: If C is the boundary of a 'nice' region $\mathcal{S}$,

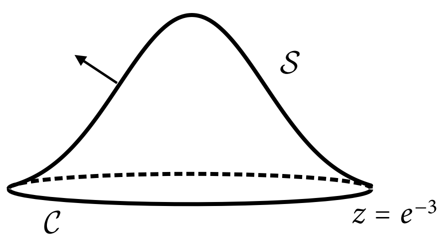
$$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS = \int_C \mathbf{F} \cdot d\mathbf{r}$$

so long as the normal $\mathbf{n}$ and the orientation of C are compatible.

In the two problems below you will set up, but **not evaluate** the integrals on both side of this equation where $\mathcal{S}$ is the surface $z = e^{-3(x^2+y^2)}$ with $x^2 + y^2 \leq 1$ and where

$$\mathbf{F} = \langle -y, x, 1 \rangle .$$

The surface is given the orientation with unit normal pointing in the direction given in the figure (generally in the positive z direction.)



1. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$. Please be careful about orientation/sign.

$$\vec{r}(t) = \langle \cos(t), \sin(t), e^{-3} \rangle$$

$$\vec{r}\,'(t) = \langle -\sin(t), \cos(t), 0 \rangle$$

$$\vec{F}(\vec{r}(t)) = \langle -\sin(t), \cos(t), 1 \rangle$$

$$\int_0^{2\pi} \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt = \int_0^{2\pi} \sin^2(t) + \cos^2(t) = \boxed{2\pi}$$

Recall: $\mathcal{S}$ is the surface $z = e^{-3(x^2+y^2)}$ with $x^2+y^2 \le 1$ and where and unit normal pointing generally in the positive z direction and that

$$\mathbf{F} = \langle -y, x, 1 \rangle .$$

2. Compute $\iint_{\mathcal{S}} (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$.

$$\nabla \times \vec{F} = \langle 0, 0, 2 \rangle$$

$$\vec{r}(u,v) = \langle u, v, e^{-3(u^2+v^2)} \rangle$$

$$\vec{r}_u = \langle 1, 0, -6u\, e^{-3(u^2+v^2)} \rangle$$

$$\vec{r}_v = \langle 0, 1, -6v\, e^{-3(u^2+v^2)} \rangle$$

$$\vec{r}_u \times \vec{r}_v = \langle 6e^{-3(u^2+v^2)}u, 6e^{-3(u^2+v^2)}v, 1 \rangle$$

$$\vec{\nabla} \times \vec{F} \cdot \vec{r}_u \times \vec{r}_v = 2 \quad (!)$$

$$\iint_R 2\, dA = 2\int_0^{2\pi}\int_0^1 r\, dr\, d\theta = 2\int_0^{2\pi} \frac{1}{2}\, d\theta$$

$$= \boxed{2\pi}$$