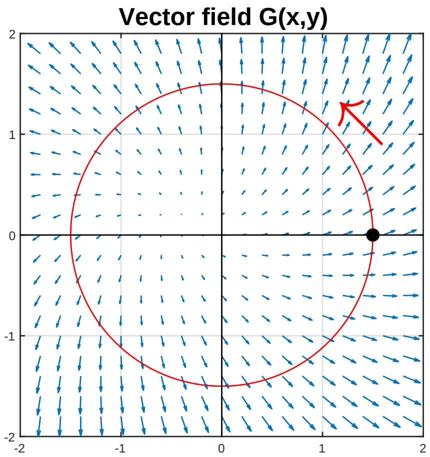
MATH 253 Name: ______________________________

'Big' Quiz 12 April 19, 2022

Instructions: (15 points total) Show all work for credit. You may use a single formula sheet which should be handed in with your quiz.

1. (3 pts.) Consider the 2-dimensional vector field $\mathbf{G}(x, y)$ shown to the left below:



Simple closed curve C with (positive) counter-clockwise orientation shown in red

Is the vector field $\mathbf{G}(x, y)$ conservative or not? Explain briefly.

2. (5 pts.) Consider the 2-dimensional vector field

$$\mathbf{F}(x, y) = \langle 2x + y,\, x + 3y \rangle .$$

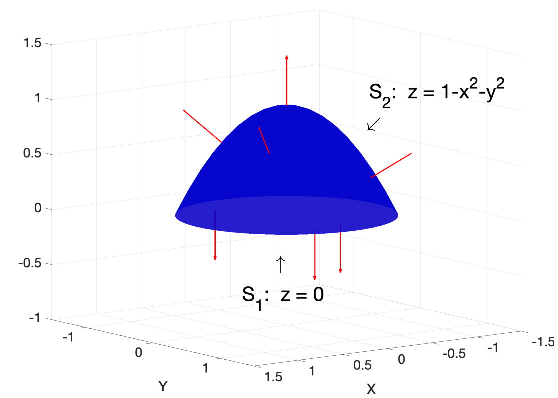
Find the work done by the vector field $\mathbf{F}$ in moving a particle along the line segment from $P(1, 1)$ to the $Q(2, 0)$.

3. (7 pts.) Consider the electrical field

$$\mathbf{E}(x, y, z) = \langle y,\, x,\, z \rangle\,.$$

By Gauss' Law, the net charge enclosed by a closed surface equals the electical flux through the surface S:

$$\text{Net charge enclosed by } S \ = \epsilon_0 \iint_S \mathbf{E} \cdot d\mathbf{S}.$$



Find the value of the flux integral across the surface S bounded by $z = 1 - x^2 - y^2$ and the xy-plane as directed. Let $S = S_1 \cup S_2$ as shown in the figure. Some normal vectors to the surface S are shown in red.

(a) (2 pts.) Carefully and succinctly justify that

$$\epsilon_0 \iint_{S_1} \mathbf{E} \cdot d\mathbf{S} = 0$$

by considering the surface S_1 (disk in xy-plane defined by $z = 0$) and the electrical field $\mathbf{E}$.

Answer: The flux integral through S_1 is zero because .....

(b) (5 pts.) From (a) and Gauss' Law, you now know that the net charge enclosed by S is

$$\epsilon_0 \iint_{S_2} \mathbf{E} \cdot d\mathbf{S}.$$

Compute this flux integral. (Next page is blank for additional work.)