Name: Solutions

1. Let $\mathcal{E}$ be the 3-d region bounded determined by the inequalities $x^2 + y^2 \leq 4$ and $0 \leq z \leq x + 2$.

a. Write down an iterated integral in terms of x, y and z variables that is equivalent to

$$\iiint_{\mathcal{E}} z\, dV.$$

Do NOT compute the value of the integral.

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{x+2} z \, dz\, dy\, dx$$

b. Write down an interated intergral in terms of cylindrical coordinates r, θ and z that is equivalent to the integral from part **a**. Do NOT compute the value of the integral.

$$\int_{0}^{2\pi} \int_{0}^{2} \int_{0}^{r\cos\theta + 2} z\, r \, dz\, dr\, d\theta$$

2. Consider the **upper half** sphere $\mathcal{E}$ given by $z \geq 0$ and $x^2 + y^2 + z^2 \leq 1$.

a. Write down an iterated integral in **spherical coordinates** that could be used to compute the value of

$$\iiint_{\mathcal{E}} z \, dV.$$

$$\int_0^{2\pi} \int_0^{\pi/2} \int_0^1 \rho \cos\phi \; \rho^2 \sin\phi \; d\rho \, d\phi \, d\theta$$

b. Compute the value of the integral. You might find a substitution is helpful to deal with the ϕ variable.

$$\int_0^{2\pi} \int_0^{\pi/2} \frac{\rho^4}{4}\Big|_0^1 \cos\phi \sin\phi \, d\phi \, d\theta$$

$$= \frac{2\pi}{4} \int_0^{\pi/2} \cos\phi \sin\phi \, d\phi$$

$$= \frac{\pi}{2} \int_0^1 u \, du \qquad u = \sin\phi$$

$$= \frac{\pi}{2} \frac{u^2}{2}\Big|_0^1 = \boxed{\frac{\pi}{4}}$$