

Name: Solutions

1. Show that the point $P(1, 2, 3)$ lies on the plane defined by $2x + 3y - z = 5$.

$$2 \cdot 1 + 3 \cdot 2 - 3 = 2 + 6 - 3 = 5 \quad \checkmark$$

2. Find the "parametric equation" of the line that passes through $P(1, 2, 3)$ and is perpendicular to the plane from problem 1.

Normal to plane: $\langle 2, 3, -1 \rangle$

Line:

$$\langle 1, 2, 3 \rangle + t \langle 2, 3, -1 \rangle$$

$$= \langle 1 + 2t, 2 + 3t, 3 - t \rangle$$

3. Find a vector perpendicular to the vectors $\mathbf{v} = \langle 1, 2, 1 \rangle$ and $\mathbf{w} = \langle 3, 1, 1 \rangle$.

$$\begin{aligned} \vec{v} \times \vec{w} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 3 & 1 & 1 \end{vmatrix} = (2-1)\hat{i} - (1-3)\hat{j} + (1-6)\hat{k} \\ &= \hat{i} + 2\hat{j} - 5\hat{k} \end{aligned}$$

4. Find the equation of a plane that passes through the points $O(0, 0, 0)$, $P(1, 2, 1)$ and $Q(3, 1, 1)$.

$$\begin{aligned} \vec{OP} &= \langle 1, 2, 1 \rangle \\ \vec{OQ} &= \langle 3, 1, 1 \rangle \end{aligned} \quad \left. \vphantom{\begin{aligned} \vec{OP} \\ \vec{OQ} \end{aligned}} \right\} \vec{OP} \times \vec{OQ} = \langle 1, 2, -5 \rangle$$

from problem 3,

Plane: $1 \cdot x + 2 \cdot y - 5z = 0$ ← plane thru origin

5. Find the equation of a plane that is parallel to the plane you found in problem 4 but that passes through the point $R(5, 1, 0)$.

Same normal, different point.

$$1 \cdot (x - 5) + 2(y - 1) - 5(z - 0) = 0$$