MATH 253 Name: ______________________________

Midterm 2 March 23, 2021

Instructions: 100 points total. Use only your brain and writing implement. You have 90 minutes to complete this exam. Good luck.

1. (8 pts.) Prove that the following limit does **NOT** exist.

$$\lim_{(x,y)\to(0,0)} \frac{4x^2y}{x^4+y^2}$$

2. (8 pts.) Find the directional derivative of $f(x,y) = xy$ at the point $P(1,9)$ in the direction from P to $Q(4,5)$. Is $f(x,y)$ (circle one) increasing / decreasing / stationary at P?

3. (8 pts.) Suppose that

$$f(x,y) = x\,e^{xy} \quad \text{where } x = t^2,\ y = \ln(t).$$

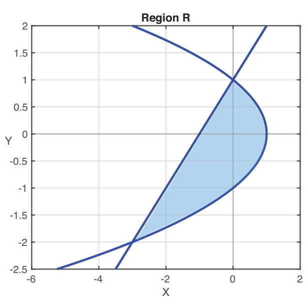
Use the **Chain Rule** to find the derivative $\frac{df}{dt}$. Simplify your answer completely for full credit and make sure it is a function only of the variable t.

4. (12 pts.) Consider the surface defined by $h(x, y) = 5x^2 + 3y^2$.

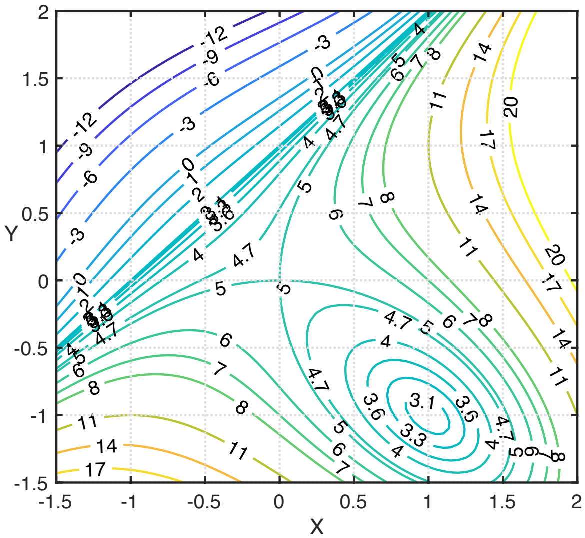
   (a) Find the tangent plane to the surface $h(x, y) = 5x^2 + 3y^2$ at the point $(1, 1, h(1, 1))$.

   (b) Estimate the value $h(.9, 1.01)$ using differentials. (Full credit only for using a linear approximation.)

5. (12 pts.) The shaded lamina (plate or region) R below is bounded by the curves with equations $y^2 = 1 - x$ and $y = x + 1$. On this lamina, the charge density is given by $\sigma(x, y) = xy$ coulombs/m^2. Find the total charge of the lamina, including units in your final answer.



6. (14 pts.) Pictured is a contour plot for the function $f(x, y) = 5 + 2x^3 - 2y^3 + 6xy$



(a) The function $f(x, y)$ has **two** local extrema at points (a, b), [i.e. a saddle point, a local maximum, or a local minimum at (a, b)]. In the table below, give the values of these extrema and the points at which they occur. Then briefly justify your answer.

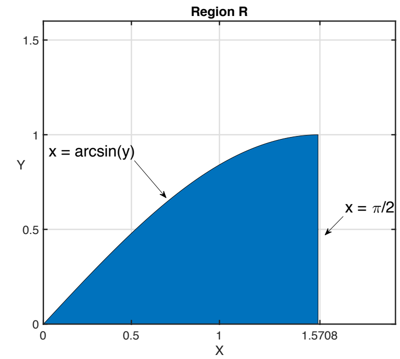
coordinates (a, b)	Value $f(a, b)$	min, max or saddle ?
1.		
2.		

Justification:

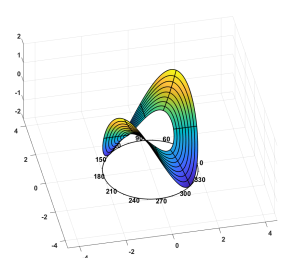
(b) Use the second derivatives test to verify your answer. That is, find all critical points of $f(x, y)$ and classify them as local maxima, local minima, or saddle points.

7. (12 pts.) Compute the double integral over the region R of integration by **reversing the order of integration.**

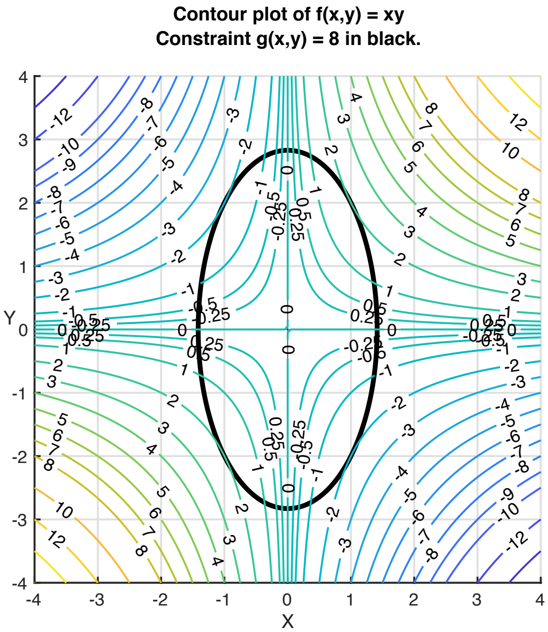
$$\int_0^1 \int_{\arcsin(y)}^{\frac{\pi}{2}} \cos(x)\sqrt{3+\cos^2(x)}\, dx\, dy$$



8. (12 pts.) Find the surface area of the part of the saddle $z = x^2 - y^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. (A picture is included for help with visualization, but is unnecessary.)



9. (14 pts.) Consider the function $f(x,y) = xy$ and its contour plot shown below.



(a) The function $f(x,y)$ has two local minima subject to the constraint $g(x,y) = 4x^2 + y^2 = 8$. (The constraint $g(x,y) = 8$ is plotted in black in the figure.) By examining the contour plot give the coordinates of the two local minima (a,b) and the value $f(a,b)$ at those points.

	(a,b)	Minimum value $f(a,b)$
1.	$(a_1, b_1) =$	
2.		

(b) Give the equations you must solve simultaneously in order to use the method of Lagrange multipliers to find the minimum values of $f(x,y)$ subject to the constraint $4x^2 + y^2 = 8$. (Be careful; it might be easy to leave out one equation.)

(c) Now verify that the first point, call its coordinates (a_1, b_1), in your list from part (a) satisfies these equations.

(d) One of the equations you gave in (b) should involve the gradient vector ∇f. Compute the gradient vectors $\nabla f(a_1, b_1)$ and $\nabla g(a_1, b_1)$, then plot them (up to a positive scaling factor) in the contour plot above. Then in the space to the right, explain briefly why the method of Lagrange multipliers works.

$\nabla f(a_1, b_1) =$

$\nabla g(a_1, b_1) =$

Explanation: