Now let's go back to $P = \frac{0.082\,T}{V}$.

Suppose $T = 300\,K$, $V = 20\,\ell$.

So $P = 1.23$ atm.

$$\frac{\partial P}{\partial V} = -0.0615 \ \frac{\text{atm}}{\ell}$$

$$\frac{\partial P}{\partial T} = \frac{0.082}{V} = 0.0041 \ \frac{\text{atm}}{K}$$

If we increase T by $\Delta T = 8\,K$,

$$\Delta P \approx 0.0041 \cdot 8 = 0.0328 \text{ atm}$$

If we increase V by $\Delta V = 2\,\ell$,

$$\Delta P \approx -0.123 \text{ atm}$$

Now: What if we did both: $\Delta T = 8K$, $\Delta V = 2\,\ell$.

$$\Delta P = \frac{\partial P}{\partial T}\Delta T + \frac{\partial P}{\partial V}\Delta V \quad ?$$ Just add both effects?

$$= -0.123 + 0.0328 = -0.0902$$

$$P\Big|_{\substack{300K \\ 20\,l}} = 1.23 \text{ atm}$$

$$P\Big|_{\substack{308K \\ 22\,l}} = \frac{0.082 \cdot 308}{22} = 1.148$$

$$\underbrace{1.23 - 0.0902}_{P_{est.}} = 1.140$$ not bad!

We codify what we just did in terms of the language of differentials

$$dP = \frac{\partial P}{\partial V} dV + \frac{\partial P}{\partial T} dT$$

Idea: you plug in dV and dT as small changes, and dP is the resulting small change in pressure.

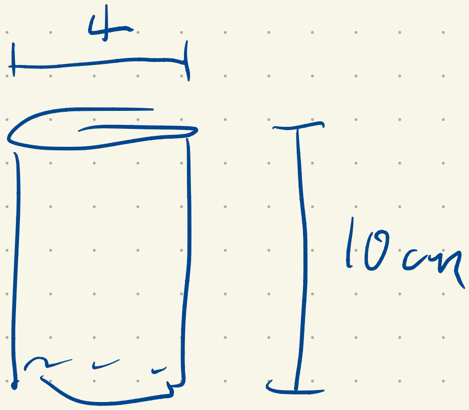
More generally:

$$z = f(x,y)$$

$$dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

e.g. Volume of a can

0.1 cm thick



$r = 2\,cm$
$h = 10\,cm$

$dr = 0.1\,cm$
$dh = 0.1\,cm$

$$V = \pi r^2 h$$

$$dV = 2\pi r h\,dr + \pi r^2\,dh$$

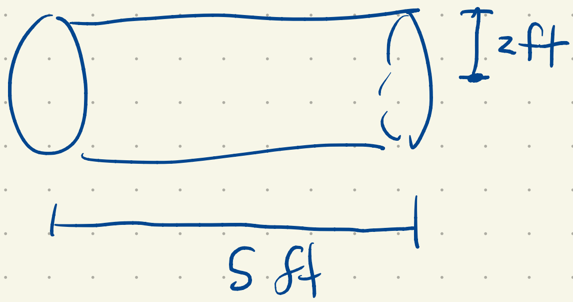
$$= \left[2\pi \cdot 2 \cdot 10 + \pi \cdot 4\right] 0.1$$

$$= \pi \left[44\right] 0.1$$

$$= \pi \cdot 4.1$$

The approximation isn't accurate, but you can see what contributes.

Suppose we have a steel tank



Is the volume more sensitive to error in the height or the radius?

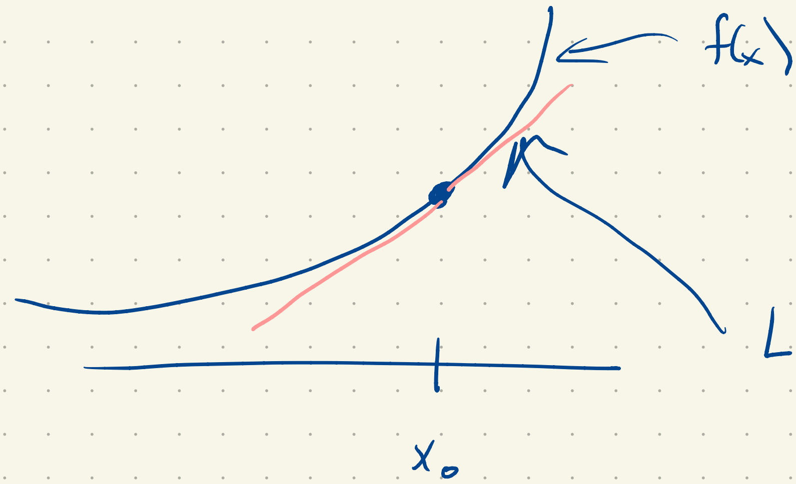
$$V = \pi r^2 h$$

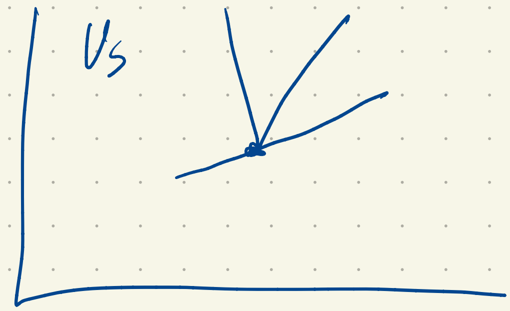
$$dV = 2\pi r h\, dr + \pi r^2 dh$$

$$= \underbrace{20\pi}\, dr + 4\pi\, dh$$

more sensitive to the radius.

Related notion: linear approximation.

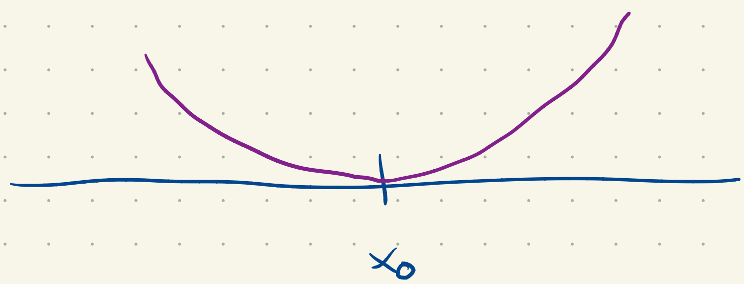




$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

Point $f(x) \approx L(x)$ for x near x_0.

$f(x) - L(x)$



error goes to 0 faster than a liner function.

For a function of two variables, at (x_0, y_0)

$$L(x,y) = f(x_0,y_0) + \left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)}(x - x_0) + \left.\frac{\partial f}{\partial y}\right|_{(x_0,y_0)}(y - y_0)$$

e.g. $V = \pi r^2 h$ linearize at $r_0 = 2$, $h_0 = 5$

$V = 20\pi$

$$L(r,h) = V(r_0,h_0) + \frac{\partial V}{\partial r}(r-r_0) + \frac{\partial V}{\partial h}(h-h_0)$$

$$= 20\pi + 4\pi \cdot 2 \cdot 5\,(r-2) + \pi r_0^2 (h-h_0)$$

$$= 20\pi + 40\pi(r-2) + 4\pi(h-5)$$

A function $f(x,y)$ is differentiable at x_0, y_0 if its linearization is a good approximation at x_0, y_0.

Good means the error

$f(x,y) - L(x,y)$ goes to 0 faster than $\sqrt{(x-x_0)^2 + (y-y_0)^2}$

e.g: $f(x,y) = x^2 + 3y^2$

At $(2,1)$

$\frac{\partial f}{\partial x} = 4 \qquad \frac{\partial f}{\partial y} = 6$

$$L(x,y) = f(2,1) + 4(x-2) + 6(y-1)$$

Plot $f(x,y) - L(x,y)$

$$f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x+y}} & (x,y) \neq (0,0) \\ 0 & (x,y) = 0,0 \end{cases}$$

$f_x = f_y = 0$ at 0.

$L(x,y) = 0$.

Plot it.

If f_x and f_y exist and are cts near $x_0 y_0$

then f is diff:

Its linearization does a good job