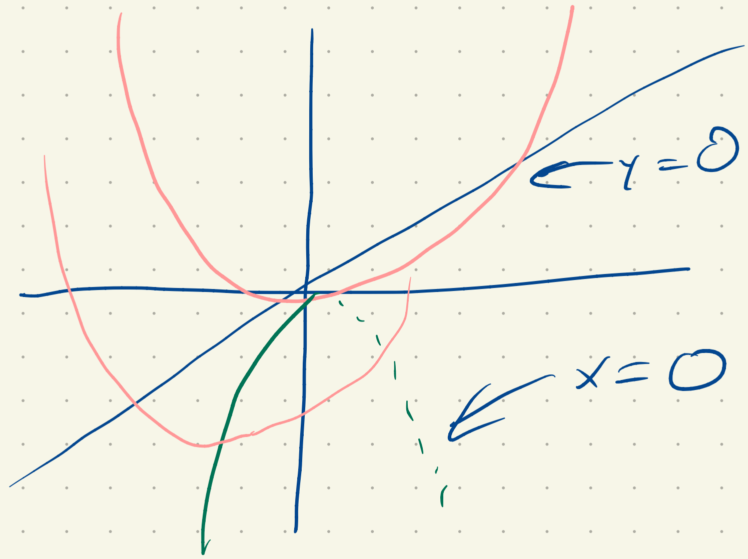


If $y=1$,

$$z=x^2-1$$

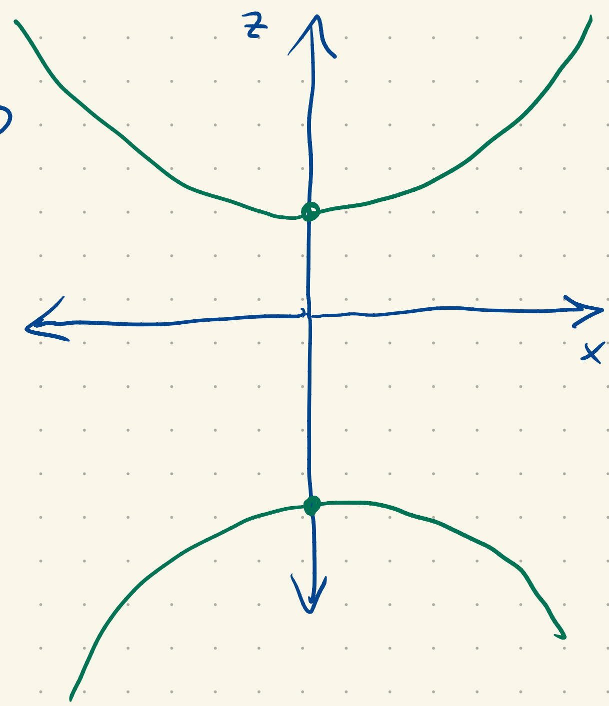
These make saddles.

Cousins: Hyperboloids

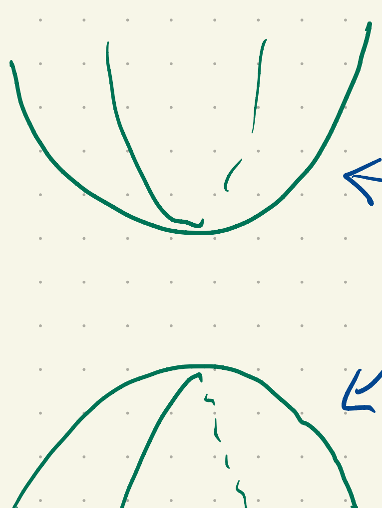
$$z^2-x^2-y^2=1$$

$z-x$ plane : $y=0$

$$z^2-x^2=1$$



Picture in $z-y$ plane is the same



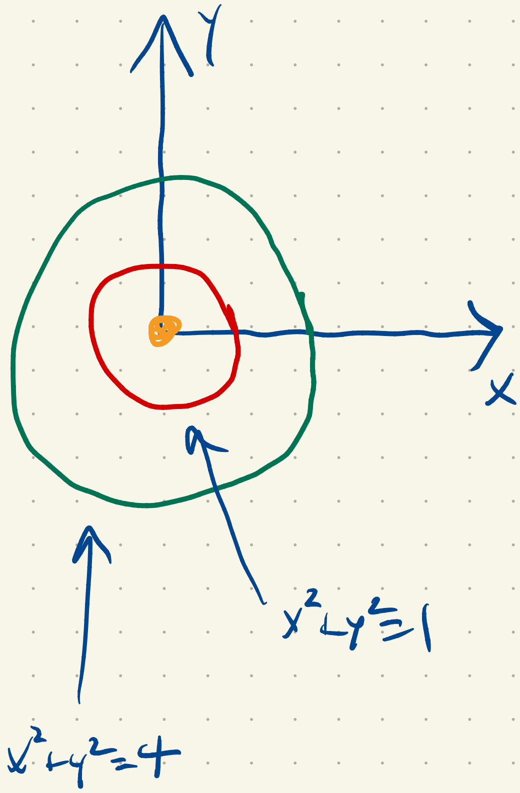
two sheeted hyperboloid

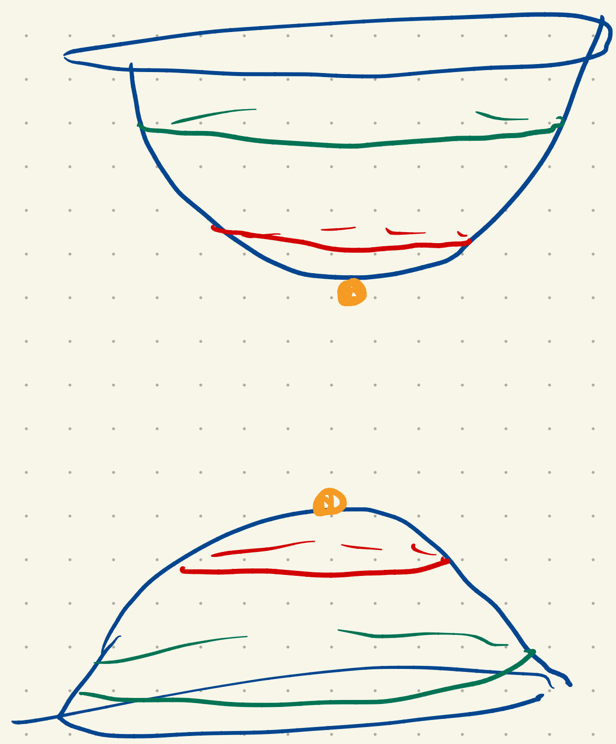
Another way to think about A

$$z^2 - x^2 - y^2 = 1$$

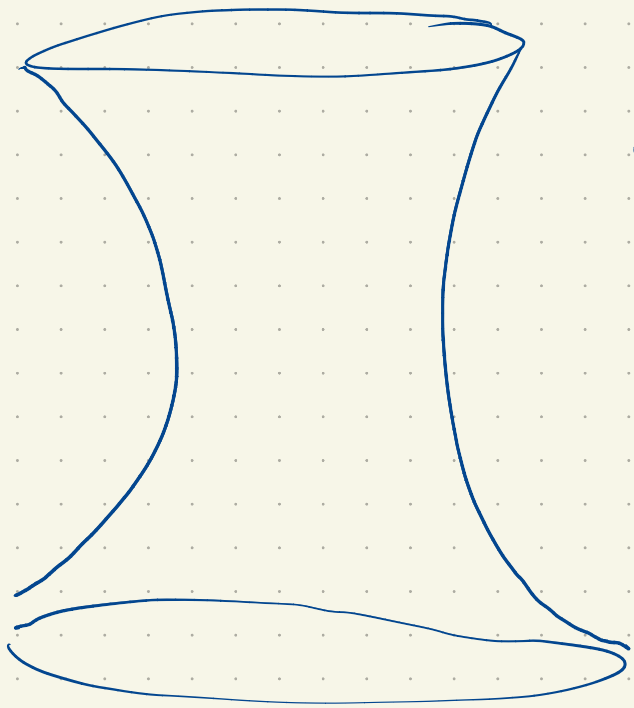
$$z = \pm\sqrt{1 + x^2 + y^2}$$

z coord only depends on x^2+y^2

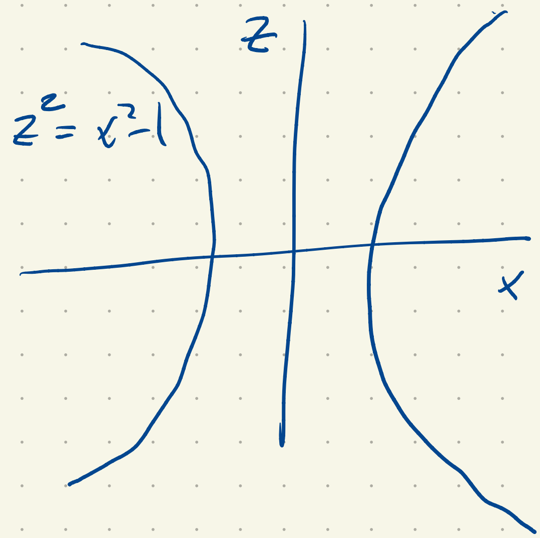




$z^2 = x^2+y^2-1$ $\quad$ ($x^2+y^2<1$ is impossible



one-sheeted hyperboloid.

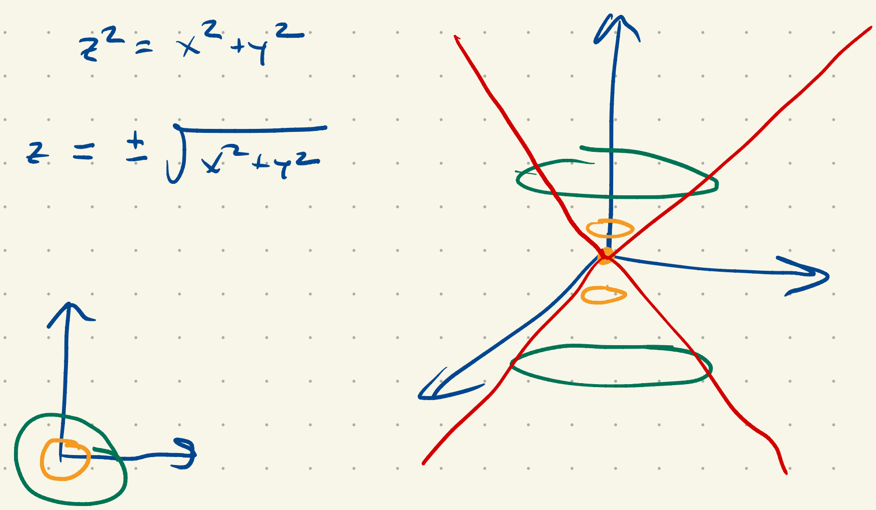


$z^2 = x^2+y^2$

$z^2 = x^2+y^2$ is a cone!

$z^2 = x^2+y^2$

$z = \pm\sqrt{x^2+y^2}$



cone (degenerate hyperboloid)

3.1

Vector-valued functions (a.k.a space curves)

In calc 1 you studied $y=f(x)$ (one input, one output)

But the real world is more complicated.

Position as a function of time has three outputs (x,y,z) depending on t

We can encode all these as

$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$$

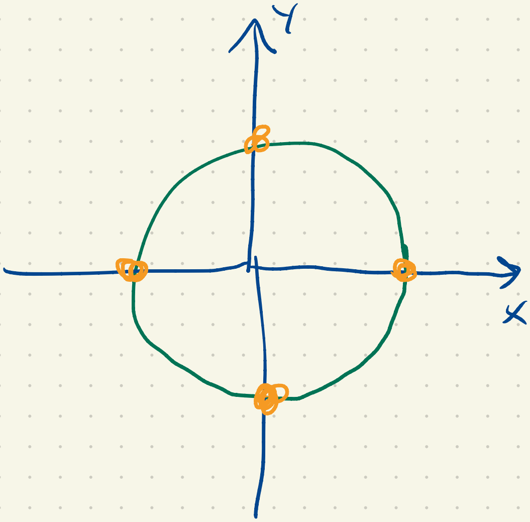
e.g. $\vec{r}(t) = \langle \cos(t), \sin(t), 0 \rangle$

$x^2+y^2=1$

$z=0$ always.

$t=0$ $\langle 1,0,0 \rangle$ $t=\pi$ $\langle -1,0,0 \rangle$

$t=\pi/2$ $\langle 0,1,0 \rangle$ $t=\frac{3\pi}{2}$ $\langle 0,-1,0 \rangle$



This a parameterized circle. It's not just a circle. It contains information about <u>when</u>.

What about $\vec{r}(t) = \langle \cos(2t), \sin(2t), 0 \rangle$

$x^2+y^2=1$ still

$t=0 \quad \langle 1,0,0 \rangle$

$t=\frac{\pi}{4} \quad \langle 0,1,0 \rangle$

$t=\pi/2 \quad \langle -1,0,0 \rangle$

$t=3\pi/4 \quad \langle 0,-1,0 \rangle$

$t=\pi \quad \langle 1,0,0 \rangle$

This parametrizes the same curve.

But: it traverses it at twice the speed!

(one rotation $0 \le t \le \pi$ not $0 \le t \le 2\pi$)

We've already seen something like this

$$\vec{r}(t) = \overbrace{\vec{r}_0}^{\langle x_0, y_0, z_0 \rangle} + \underbrace{\vec{v}}_{\langle a,b,c \rangle} t = \langle x_0 + at,\ y_0 + bt,\ z_0 + ct \rangle$$

This is a parameterized line. It's not just the line

One more example:

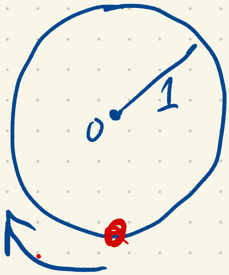
$$\vec{r}(t) = \langle \cos t,\ \sin(t),\ 2t \rangle$$

Now it doesn't stay in the x-y plane.

As t progresses, z increases steadily

Helix!

e.g.

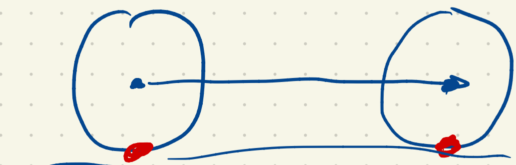


$\theta(t) = -\pi/2 - t$

$\langle \cos(-\frac{\pi}{2}-t), \sin(-\frac{\pi}{2}-t) \rangle$

$\langle \cos(t+\pi/2), -\sin(t+\frac{\pi}{2}) \rangle$

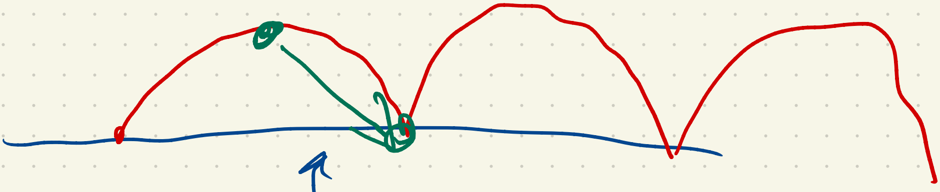
$\langle -\sin(t), -\cos(t) \rangle$



center: $\langle t, 1 \rangle$

x, y in cm

cycloid: $\langle t-\sin(t), 1-\cos(t) \rangle$



displacement!

Q: what is the displacement of the dot between time $t=\pi$ and $t=2\pi$

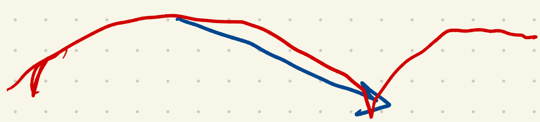
$$\vec{r}(\pi) = \langle \pi, 1+1 \rangle = \langle \pi, 2 \rangle$$

$$\vec{r}(2\pi) = \langle 2\pi, 1-1 \rangle = \langle 2\pi, 0 \rangle$$

$$\vec{r}(2\pi) - \vec{r}(\pi) = \langle \pi, 2 \rangle - \langle 2\pi, 0 \rangle$$

$$= \langle -\pi, 2 \rangle \text{ cm}$$

$$\vec{r}(t_2) - \vec{r}(t_1)$$



Q: what is the average velocity over the same time period?

$\frac{\text{displacement}}{\text{time}}$ $\qquad \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$

So in this case $\frac{1}{\pi}\langle -\pi, 2 \rangle$

$\langle -1, \frac{2}{\pi} \rangle \frac{cm}{s}$